Newberger Math 247 Spring 03

Solutions for Quiz 1.5 and 1.7

$$A = \begin{bmatrix} 1 & 2 & -1 \\ -2 & 8 & -10 \\ 5 & -4 & 9 \\ 5 & 2 & 3 \end{bmatrix} \; b = \begin{bmatrix} 0 \\ -12 \\ 14 \\ 8 \end{bmatrix}$$

1. (a) (8 points) Solve the system $A\mathbf{x} = \mathbf{b}$ where A and $\mathbf{b}$ are given above. Put your answer in parametric vector form.

To solve, reduce the augmented matrix.

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ -2 & 8 & -10 & -12 \\ 5 & -4 & 9 & 14 \\ 5 & 2 & 3 & 8 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 12 & -12 & -12 \\ 0 & -14 & 14 & 14 \\ 1 & -8 & 8 & 8 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Note that there are three columns, so the vector $\mathbf{x}$ *is in* $\mathbb{R}^3$.

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 - x_3 \\ -1 + x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} x_3$$

(b) (4 points) Give a geometric description of the solution set that you found in part (a).

The solution set to this matrix equation is a line in $\mathbb{R}^3$ *passing through the vector* $\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$ *parallel to the span of the vector* $\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$.

2. (8 points) Determine whether or not the columns of A are linearly independent, where the matrix A is given above.

To answer this question, we need to see whether the homogeneous system $A\mathbf{x} = \mathbf{0}$ *has only trivial solution (in which case the columns are independent) or non-trivial solutions (in which case the columns are dependent). You may reduce the homogeneous system, if you'd like, but you have already done the work in number (1). Since the matrix does not have a pivot in every column, there are free variables and hence infinitely many solutions. So the homogeneous equation has non-trivial solutions, and as a result we know the columns of A are linearly dependent.*

That completely answers the quiz question, but we can say more. The reduced augmented matrix for the homogeneous equation looks like

$$\left[\begin{array}{rrr|r} 1 & 2 & -1 & 0 \\ -2 & 8 & -10 & 0 \\ 5 & -4 & 9 & 0 \\ 5 & 2 & 3 & 0 \end{array}\right] \sim \left[\begin{array}{rrr|r} 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

The third column is not a pivot column, so it is linearly dependent on the first two columns. Suppose we were asked to express the non-pivot columns in terms of the pivot columns. From the reduced matrix we see that the weights for expressing the third column in terns of the first are 1 *and* -1*. So we get*

$$\begin{bmatrix} -1 \\ -10 \\ 9 \\ 3 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ -2 \\ 5 \\ 5 \end{bmatrix} + (-1) \begin{bmatrix} 2 \\ 8 \\ -4 \\ 2 \end{bmatrix}.$$