

Quiz Solutions 1.1 and 1.2

1. Consider the following system.

$$\begin{aligned}x_1 + x_2 + 4x_3 &= -2 \\ -2x_1 - 2x_2 + hx_3 &= k \\ 2x_2 + 8x_3 &= 0\end{aligned}$$

Find all h and k such that the system has (a) no solution, (b) a unique solution, and (c) an infinite number of solutions.

Reduce the augmented matrix for the system to echelon form.

$$\left(\begin{array}{ccc|c} 1 & 1 & 4 & -2 \\ -2 & -2 & h & k \\ 0 & 2 & 8 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 4 & -2 \\ 0 & 2 & 8 & 0 \\ 0 & 0 & h+8 & k-4 \end{array} \right).$$

- (a) *This system has no solution when $h = -8$ and when k is any real number except 4 (because this will put a pivot position in the last column of the augmented matrix).*
 (b) *It has a unique solution when k is any real number, and h is any real number except -8 (because this will put a pivot position in every column of the coefficient matrix).*
 (c) *It has an infinite number of solutions when $h = -8$ and $k = 4$ (because then it will have a column that does not have a pivot position, and hence a free variable).*

2. Find the general solution of the following system of equations.

$$\begin{aligned}x_2 + 3x_3 &= -2 \\ -2x_1 - 3x_2 - 5x_3 &= -4 \\ x_1 + 2x_2 + 4x_3 &= 1\end{aligned}$$

We write the augmented matrix for this system and reduce it to reduced echelon form.

$$\begin{aligned}& \left(\begin{array}{ccc|c} 0 & 1 & 3 & -2 \\ -2 & -3 & -5 & -4 \\ 1 & 2 & 4 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 4 & 1 \\ 0 & 1 & 3 & -2 \\ -2 & -3 & -5 & -4 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 4 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 1 & 3 & -2 \end{array} \right) \\ & \sim \left(\begin{array}{ccc|c} 1 & 2 & 4 & 1 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -2 & 5 \\ 0 & 1 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right).\end{aligned}$$

So the basic variables are x_1 and x_2 , and x_3 is free. The equations are:

$$\begin{aligned}x_1 - 2x_3 &= 5 \\ x_2 + 3x_3 &= -2\end{aligned}$$

In general form the solution set is given by:

$$\begin{aligned}x_1 &= 2x_3 + 5 \\ x_2 &= -3x_3 - 2 \\ x_3 &\text{ is free.}\end{aligned}$$