Newberger Math 247 Spring 03

Homework solutions: Section 2.8 #31-36

31. Suppose F is a 5×5 matrix whose column space is not equal to $\mathbb{R}^5$. What can you say about Nul F?

Start your explanations with the assumptions (the suppose part). Since the column space of F is not equal to $\mathbb{R}^5$, F does not have a pivot in every row. Since F is square, we know F also does not have a pivot in every column. This means that the homogeneous equation $F\mathbf{x} = \mathbf{0}$ has non-trivial solutions. Since Nul F is the solution set to the homogeneous equation, we can deduce that Nul F contains nonzero vectors, or in other words, Nul $F \neq \{\mathbf{0}\}$.

32. If R is a 6×6 matrix and Nul R is not the zero subspace, what can you say about Col R?

Start your explanation with the assumptions (the if part). Since Nul R is not the zero subspace, we know that the homogeneous equation $R\mathbf{x} = \mathbf{0}$ has nontrivial solutions. This means that R does not have a pivot in every column. Since R is square, this also means that R does not have a pivot in every row, so the columns of R do not span all of $\mathbb{R}^6$. Thus Col R is not equal to $\mathbb{R}^6$.

33. If Q is a 4×4 matrix and Col $Q = \mathbb{R}^4$, what can you say about solutions of equations of the form $Q\mathbf{x} = \mathbf{b}$ for $\mathbf{b}$ in $\mathbb{R}^4$?

Start with the assumptions (the if part). Since Col $Q = \mathbb{R}^4$, the span of the columns of Q is all of $\mathbb{R}^4$, so Q has a pivot in every row. Furthermore, since Q is square, Q also has a pivot in every column. This means that $Q\mathbf{x} = \mathbf{b}$ has a unique solution for all $\mathbf{b}$ in $\mathbb{R}^4$. Keep straight in your mind that a pivot in every row means the solution always exists, and a pivot in every column means that when there is a solution, that solution is unique.

34. If P is a 5×5 matrix and Nul P is the zero subspace, what can you say about solutions of the equations of the form $P\mathbf{x} = \mathbf{b}$ for $\mathbf{b}$ in $\mathbb{R}^5$?

Start with the assumptions (the if part). Since Nul P is the zero subspace, the homogeneous equation $P\mathbf{x} = \mathbf{0}$ has only the trivial solution, which means that P has a pivot position in every column. Since P is square, P also has a pivot in every row. Thus $P\mathbf{x} = \mathbf{b}$ has a unique solution for every $\mathbf{b}$ in $\mathbb{R}^5$. Keep straight in your mind that a pivot in *every row means the solution always exists, and a pivot in every column means when there is a solution, that solution is unique.*

35. What can you say about Nul B when B is a 5×4 matrix with linearly independent columns?

Since B has linearly independent columns, B has a pivot in every column. This means that the homogeneous equation $B\mathbf{x} = \mathbf{0}$ has only the trivial solution, so Nul $B = \{\mathbf{0}\}$ where $\mathbf{0}$ denotes the zero vector in $\mathbb{R}^4$.

36. What can you say about the shape of an $m \times n$ matrix A when the columns of A form a basis for $\mathbb{R}^m$?

If the columns of A form a basis for $\mathbb{R}^m$, the columns of A must span $\mathbb{R}^5$ (so A must have a pivot in every row), and the columns must be linearly independent (so A must have a pivot in every column). Thus in this case $m = n$. Thus A is a square matrix.