

Homework solutions: Section 1.2, #23-26, 29, 30

23. Suppose that a 3×5 coefficient matrix for a system has three pivot columns. Is the system consistent?

This system is consistent. The coefficient matrix has 3 rows. Since the coefficient matrix has 3 pivots, there is a pivot in each row. This means each row in the coefficient matrix has a non-zero entry. So in the augmented matrix there cannot be a row of the form

$$[0 \ 0 \ 0 \ 0 \ 0 \ b]$$

where $b \neq 0$. By Theorem 2, the system is consistent.

24. Suppose that a system of linear equations has a 3×5 augmented matrix whose fifth column is a pivot column. Is the system consistent?

This system is inconsistent. The fifth column of the augmented matrix is the rightmost column and since it is a pivot column, the matrix has a row of the form

$$[0 \ 0 \ 0 \ 0 \ b]$$

where $b \neq 0$. By Theorem 2, the system is inconsistent.

25. Suppose the coefficient matrix of a system of linear equations has a pivot position in every row. Explain why the system is consistent.

The coefficient matrix has a pivot in every row. This means each row in the coefficient matrix has a non-zero entry. So there can be no row in the augmented matrix of the form

$$[0 \ \dots \ 0 \ b]$$

where $b \neq 0$. By Theorem 2, the system is consistent.

26. Suppose the coefficient matrix of a linear system of three equations in three variables has a pivot in each column. Explain why the system has a unique solution.

In the coefficient matrix, there are 3 rows and 3 pivots, so there must be a pivot in each row, making the system consistent (as in #25). Since there is a pivot in each column of the coefficient matrix, all of the variables are basic variables, which means there are no free variables and the solution is unique.

29. A system of linear equations with fewer equations than unknowns is sometimes called an underdetermined system. Suppose that such a system happens to be consistent. Explain why there must be an infinite number of solutions.

The number of pivot positions is less than or equal to the number of rows in the matrix. Thus an underdetermined system has fewer

pivot positions than variables. This means not all variables are basic variables, and hence the system must have free variables. Thus since it is consistent, it will have infinitely many solutions.

30. Give an example of an inconsistent underdetermined system of two equations in three unknowns.

Note that the question asks for equations, not a matrix! One way to make an inconsistent system is to use the same coefficients in two equations, but different answers. For example:

$$x_1 + x_2 + x_3 = -2$$

$$x_1 + x_2 + x_3 = 5.$$