Abstract Algebra, Newberger, Fall 2006
Tips and Remarks for Sections 6.4 (but it said 6.3 in the assignment)

p 157 #3: (a) Prove that a nonzero integer p is prime if and only if the ideal (p) is maximal in $\mathbb{Z}$.
(b) Let F be a field and $p(x) \in F[x]$. Prove that $p(x)$ is irreducible if and only if the ideal $(p(x))$ is maximal in $F[x]$.

Part (a) can be done by combining the if and only if statements in Theorems 2.8 and 6.15. Part (b) can be done by combining the if and only if statements in Theorems 5.10 and 6.15.

p 158 #17: Let $f : R \to S$ be a surjective homomorphism of commutative rings. If J is a prime ideal in S and $I = \{r \in R | f(r) \in J\}$, prove that I is a prime ideal in R.

This question focuses on the set $I = \{r \in R | f(r) \in J\}$. Note that by the definition of this set, $x \in I \Leftrightarrow f(x) \in J$.

Begin by proving that I is an ideal (using for example, Theorem 6.1). Then show that I is a prime ideal, being careful to prove both that (1) $I \neq R$ and that (2) if $ab \in I$, then $a \in I$ or $b \in I$.

To show (1) $I \neq R$, use that $J \neq S$ (since J is a prime ideal in S). Since $J \neq S$, you can find an element $y \in S$ such that $y \notin J$. Use that f is surjective to show that there is $x \in R$ such that $x \notin I$, thus implying that $I \neq R$. That is the only place in this exercise that you will use the surjectivity of f.