Math 444 Fall 2006
Newberger

Homework 5 Tips and Remarks: Section 4.2

I. Page 93 #3. If $a, b \in F$ and $a \neq b$, show that $x + a$ and $x + b$ are relatively prime in $F[x]$.

Let $d(x)$ be the greatest common divisor of $x + a$ and $x + b$. Then you want to prove that $d(x) = 1_F$. Look at the definition of the gcd of two polynomials; the first part says that $d(x)$ divides both $x + a$ and $x + b$. Write down the equations that arise from the definition of divides. Take the degree of one of these equations, and deduce that $deg(d(x))$ is either 0 (case 1) or 1 (case 2). Now degree 1 polynomials have the form $ax + b$ for some $a, b \in F$ and degree 0 polynomials are constant. Use that and the equations that arose from the definition of divides above to show $d(x) = 1_F$ in each case.

II. Page 93 #4.
(a) Use the definition of divides to write equations from the assumptions $f(x)\big|g(x)$ and $g(x)\big|f(x)$. Use these equations to show that $deg(f(x)) = deg(g(x))$. Then deduce that the other polynomial in each equation must be constant.

(b) Use the conclusion to part (a) and equate coefficients to deduce that the constant is 1_F.

III. Page 94 #7.

The assumption in this problem is a "for every statement." To use it, you must specify to which polynomial $g(x)$ you will apply the statement. You can pick any (nonconstant) $g(x)$ that you want, and the statement $f(x)|g(x)$ will still hold. For example, the statement will hold if $g(x) = x + 1$ and $g(x) = x + 2$. Refer to part (I) above to complete the argument.