Math 444 Fall 2006
Newberger

Homework Tips and Remarks: Section 4.1

II. B. Problem #12 page 89.

The proof (and statement) to this exercise is very similar to that of Theorem 4.8.

III. Do problem #18 page 89. In addition to answering the questions asked therein, answer this: Is D a bijection?

Let $D : \mathbb{R}[x] \to \mathbb{R}[x]$ be the derivative map defined by

$$D(a_0 + a_1 x + \cdots + a_{n-1}x^{n-1} + a_n x^n) = a_1 + \cdots + (n-1)a_{n-1}x^{n-2} + na_n x^{n-1}$$

Is D a homomorphism? Is D an isomorphism? (Extra part) Is D a bijection?

The answer to all three of these is no. In order to show D is not a homomorphism, show that $D(f(x)g(x)) = D(f(x))D(g(x))$ is false. To show a statement is false, provide a counter example. In other words, give polynomials (simple ones, with particular numerical coefficients) $f(x)$ and $g(x)$, calculate the product of their derivatives and the derivative of their product and get different answers.

To show that D is not a bijection, show that it is not one-to-one. To show a statement is false, provide a counter example. This amounts to giving two different polynomials (simple ones with numerical coefficients) that have the same derivative.