Math 444 Fall 2006
Newberger

Tips and Remarks for Homework 2: Section 3.1

Here are tips and remarks for page 51 #7b. Use these guidelines to complete #8b and #9, as well.
I.B. page 51 #7 part b. Let R be a ring and consider the subset R^* of $R \times R$ defined by $R^* = \{(r,r)|r \in R\}$. For any ring R show that R^* is a subring of $R \times R$.

Use Theorem 3.2, which says that there are four things to check in order to prove that R^* is a subring of $R \times R$.

(i) To prove that R^* is closed under addition, prove the statement: if $a, b \in R^*$, then $a + b \in R^*$.
  - Write out your assumptions (the if part): Let $a, b \in R^*$. Then there exists $r \in R$ such that $a = (r, r)$, and there exist $s \in R$ such that $b = (s, s)$. Here we are saying that if a and b belong to the set R^* they must take the form given in the definition of the set R^*.
  - Now explain what you want to prove (the then part): We want to show that $a + b \in R^*$. This means you want to find an element $t \in R$ such that $a + b = (t, t)$. Here we are saying that to prove that $a + b$ is an element of R^*, we must show that $a + b$ takes the form given in the definition of the set R^*.
  - Next, you will use the if to get the then. Use the assumptions to write $a+b$ as a single ordered pair, to determine what t must be. Say what t is and verify that t is an element of R. Conclude that $a + b \in R^*$.

(ii) Argue that R^* is closed under multiplication using a similar argument to that outlined in (i).

(iii) You are proving that R^* is a subring of $R \times R$. This means that the R in Theorem 3.2 corresponds to $R \times R$, while the S in the theorem corresponds to R^*. So you must show that $0_{R \times R} \in R^*$. Write what $0_{R \times R}$ is, and verify that it has the form given in the definition of the set R^*.

(iv) Let $a \in R^*$. Write a in the form given in the definition of the set R^*. Write what the additive inverse of a is. Explain that the additive inverse can be written in the form given in the definition of the set R^*, and is hence an element of R^*. Note that $-(r, r)$ is not in the form given in the definition of the set R^* (because of the minus sign), while $(-r, -r)$ is in that form.