Abstract Algebra, Newberger, Fall 2006

Tips and Remarks for the homework from Sections 3.3

I. Subrings. Do problem #28 page 78.

Let $f : R \to S$ be a homomorphism of rings and let $K = \{r \in R | f(r) = 0_R\}$. Prove that K is a subring of R.

Use Theorem 3.6. In proving that K is closed under subtraction and multiplication, you will use that f is a homomorphism.

Let $r, s \in K$. Then r and s have the property that defines the set K. In particular, $f(r) = 0_R$ and $f(s) = 0_R$. To prove that K is closed under subtraction, you must show that $r - s$ belongs to K, i.e. that $f(r - s) = 0_R$. Use the definition of homomorphism to show this. Use a similar argument for closure under multiplication.

II. Isomorphisms. Do problem #8 page 76.

Prove that the field $\mathbb{R}$ is isomorphic to the ring of all 2×2 matrices of the form $\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}$, where $a \in \mathbb{R}$.

Let S be the set of all 2×2 matrices of the form $\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix}$, where $a \in \mathbb{R}$. We want to make up a function $f : \mathbb{R} \to S$ that is injective, surjective and a homomorphism. When you say what your function is, it is not enough to write "Let $f : \mathbb{R} \to S$." You must say, "Let $f : \mathbb{R} \to S$ be given by $f(x) = ...$," and say what the answer is. Use the example on page 69 to guide you through the proofs.