

Newberger Math 444 Fall 2006

Final Exam Review Sheet

The final exam will be in two parts, a cumulative part, and a part covering new material, each worth half the credit for the exam.

New Material

- (1) The following list consists of terms that you may be asked to define, and theorems you may be asked to state.
 - Permutation (page 161).
 - Group (page 163).
 - Abelian group (page 163).
 - Order of a group (page 163).
 - Symmetric group on n symbols (page 164).
 - Dihedral group of degree n (page 167).
 - Order of an element (page 176).
 - Infinite order, finite order elements (page 176).
 - Subgroup (page 181).
 - Center of a group (page 183).
 - Cyclic subgroup generated by a , cyclic group (page 184).
 - Isomorphism of groups, homomorphism of groups (page 191).
 - Cayley's Theorem (page 194).
 - Congruence modulo K (a subgroup) (page 200).
 - Right coset (page 201).
 - Lagrange's Theorem (page 202).
 - Normal (page 211).
 - Quotient group G/N (page 216, 217).
- (2) You should know the statements of the following lemmas, theorems and corollaries so that you may use them in your arguments:
 - From Section 7.1: All.
 - From Section 7.2: All except: in Theorem 7.8, we only studied item (3), and we did not study Corollary 7.9.
 - From Section 7.3: Theorems 7.10, 7.12, 7.13.
 - From Section 7.4: All.
 - From Section 7.5: Theorems 7.22, 7.23, 7.24, 7.25, 7.26, 7.27
 - From Section 7.6: Theorem 7.34 (1) \Leftrightarrow (3) only.
 - From Section 7.7: Theorems 7.35, 7.36.
 - From Section 7.8: Theorem 7.39, 7.40, 7.42.
- (3) You will be asked to provide a proof of at least one of the following (the statements will be given): Theorem 7.8 (3) (the part that says $a^k = e \Leftrightarrow n|k$ only), and Theorem 7.12.

Overview of the part covering new material

- You **will** be asked to state some of the definitions and theorems listed in (1) above and to prove at least one of those listed in (3).
- You **will** be asked to prove a given subset of a group is a subgroup. You may be asked to prove it is normal.
- You **may** be asked to list the elements in a cyclic subgroup, or to list all cyclic subgroups in a group. You may be asked to show a group is or is not cyclic.
- You **may** be asked to list the elements in each distinct coset of a given subgroup.
- You **may** be asked to find the order of an element or the order of a group, or to answer questions that involve using Lagrange's Theorem.
- You **will** be asked to prove a given function is an isomorphism or a homomorphism, or to create an isomorphism between two given sets.

Overview of the cumulative part

The cumulative part of the exam covers the material listed on the review sheets for Exams 1 and 2. You may be asked to state definitions and theorems from the lists on those review sheets, but you will not be asked to prove the theorems that you memorized for those exams.

- You **may** be asked to give examples of rings, rings with or without identity, rings that are or are not commutative, integral domains, or fields.
- You **will** be asked to prove things about particular polynomials or their congruence classes. For example, that two given polynomials are relatively prime, that a given polynomial is or is not irreducible, or that the congruence class of a given polynomial is or is not a unit. These questions may involve the degree of a polynomial, and/or the division algorithm.
- You **may** be asked a question dealing with the relationships between properties of an ideal I (or a polynomial $p(x)$) and the quotient ring R/I (or $F[x]/(p(x))$). This may involve prime or maximal ideals, or irreducible polynomials.
- You **will** be asked to prove statements involving homomorphisms or isomorphisms. These may involve the kernel of a homomorphism, or its image, or the First Isomorphism Theorem.
- You **may** be asked to prove an object is unique, or to use that an object is unique to prove a statement.