

Homework Sections 6.4.

I. These exercises are derived from problem #74 page 108, problem #10 page 354, and problem #11 page 354.

A. Read problem #74 page 108. This part will guide you to write a paragraph that gives a solution to that problem. Do not number the sentences in your paragraph; they are numbered here for easy reference, but your solution should be a paragraph not a list. When you write an equation in your paragraph, like

$$y = nx^m,$$

it can be easier to read if you center it and put it on a new line, like we did here.

1. Begin by writing a sentence that explains what it means for x and y to be related by an allometric formula, and what allometric formulae are used for.
2. Write a sentence (or two) that explains what you get when you take the natural log of both sides of the equation and simplify the right hand side, using the laws of logs.
3. Now write a sentence to explain that you can consider $(\ln x)$ and $(\ln y)$ as your variables, instead of x and y . To further explain this idea, say that you will let $X = \ln x$ and $Y = \ln y$, and substitute X and Y into your equation.
4. Explain that the resulting equation with X and Y is the equation of a line. Say what the slope and intercept are.
5. Write a conclusion sentence that explains that if x and y are related by an allometric equation, then $\ln x$ and $\ln y$ are related by a linear equation.

B. Read problem #10 page 354. This part will guide you to write a paragraph that gives a solution to that problem. Again, write a paragraph, not a list. Do not number your sentences.

1. You will study quantities x and y that are related by an allometric equation, but in this case, the quantities are changing with time (so that x and y are functions of time). Write a sentence explaining this.
2. Explain that, as above, you can take the natural log of the equation and use the laws of logs to simplify the right hand side. If you want to, you can refer to your previous paragraph, and just give the result, rather than show your work again.
3. Now explain that since the quantities x and y are changing with respect to time, we can study the relationship between the rates at which each variable is changing. Explain that this means we want to find an equation having both the rate of change of x with respect to time (dx/dt) and the rate of change of y with respect to time (dy/dt).
4. Write a sentence explaining what you get when you take the derivative of both sides of the equation you obtained in step (2) above with respect to t . (Tip: be careful when you take your derivatives to use the chain rule, so that you get the terms dx/dt and dy/dt to show up.)

5. Conclude that by using the relationship between the quantities x and y (both functions of time), you found a relationship between their rates of change dx/dt and dy/dt .
- C. Read problem 11, page 354. While this exercise does not refer to the two exercises you just completed, the quantities b and w are related by an allometric formula. This part guides you to write a paragraph explaining this and using your results above to answer the question.
1. Write a sentence explaining the model given in problem 11, page 354, making note of the reference given for the research that yielded the model. Include units
 2. Write a sentence to explain that b and w are related by an allometric formula $y=nx^m$, saying what x , y , n and m are in this case.
 3. Write a sentence that explains that since x and y are related by an allometric formula, the relationship between dx/dt and dy/dt that you derived in part B holds. You do not need to derive the relationship again, just say what it is.
 4. Write a sentence that restates this relationship with the variables w and b , and rates of change dw/dt and db/dt .
 5. For the rest of the paragraph, explain that you can use the formula to find the rate of change of the total weight of the fetus, for a fetus with brain weight 25 g and brain weight growing at a rate of 0.25 grams per day. Begin by explaining what the weight of this fetus is, and how you found it.
 6. Then explain where in the formula you will plug in 25 and 0.25, and the number you found for the weight of the fetus in part (5), and say what you will be solving for.
 7. Explain the work you do solving this equation.
 8. Write a conclusion sentence that summarizes your result.

II. Read problem #16 page 354. The model in this problem comes from an article published in a journal called *Wildlife Feeding and Nutrition* in 1983. While the relationship between the variables in this problem is allometric, we will solve it using the power rule and the chain rule, rather than the formulas that you derived in the previous exercises.

- A. Write a sentence explaining what the model is, and that both variables are functions of time.
- B. Take the derivative of the equation with respect to time, making sure to use the chain rule, so that the variables dE/dt and dw/dt show up.
- C. Say where you will plug in the numbers 10 and 0.1, and say what you will solve for.
- D. Express your answer as a sentence, being careful to get the correct units.