Homework Sections 5.2.

I. The percent of concentration of drug in the bloodstream x hours after the drug has been administered is modeled by a function of the form

$$k(x) = \frac{x}{mx^2 + 1},$$

where m is a positive constant.

A. Suppose that $m = 4$.

1. Find the derivative of k(x), and its critical numbers.

2. Draw a number line, and plot the critical numbers on it, labeling them as such. Above the number line, indicate the sign of $k'(x)$ with the symbols + or -. Label the number line "sign of $f'(x)$."

3. On what interval is the percent concentration of drug in the bloodstream increasing? On what interval is the percent concentration decreasing?

4. At what time does the percent concentration of drug in the blood stream reach a maximum value?

B. Generalize your results by repeating exercise A, leaving m as a variable. When you take your derivative, remember that m is a constant. Your answers will have m's in them. (If you want to check yourself, plug $m = 4$ into your answers, and make sure you get the same results as you did in part A. Do not write these checks on your homework.)

C. When a drug is administered by injection, it enters the bloodstream more efficiently than when the drug is administered by mouth. Do you expect that the value of m when modeling a drug administered by injection to be larger or smaller than the value of m when modeling a drug administered by mouth? To answer this question, write a paragraph following the outline below.

1. Write a topic sentence that answers the question.

2. To explain your answer, give two examples of values of m, one that might be for a drug administered by mouth and one that might be for a drug administered by injection.

3. Then use the formulas you created in part B to say at what time the drug reaches its maximum percent concentration for each value of m.

4. Conclude (i.e. make sure) that your examples support your claim.

Note that when I grade part C, I will be checking that you included each of the items (1)-(3) in your explanation.

II. These questions relate to problem #8 page 288.

A. Copy the graph of the function $f(x)$ into your homework (not too small).

B. List the critical numbers for this function. For each critical number, say whether $f'(c) = 0$ or $f'(c)$ does not exist. Write sentences like: "The critical numbers are…. $f'(c)$ does not exist at $c = \ldots$, and $f'(c) = 0$ at $c = \ldots$"

C. Draw a number line, and plot the critical numbers on it, labeling them as such. Above the number line, indicate the sign of $f'(x)$ with the symbol + or -. Label the number line "sign of $f'(x)$."

D. On what intervals is $f(x)$ increasing and on what intervals is $f(x)$ decreasing?

E. Explain whether or not f(x) has each of the following, and give the coordinates (in the form (x,y)) at which they are achieved if they exist: local maxima, local minima, absolute maxima, absolute minima.

III. These questions relate to problem #42 on page 329. A tooth is made up mostly of dentin, which is a hard bone-like tissue. This research was published in a journal called, *Journal of Craniofacial Genetics and Developmental Biology*, in 1996.

A. Use Excel to graph the functions given in problems 42, for t between 9 and 51. Adjust the y-axis to show the y-values between 0 and 1.5. You should be able to find step by step instructions for the various aspects of this plot in previous assignments.

B. Look at the graph and estimate at how many days the volume of the dentin reaches its highest value. This is asking you to look for the largest value of the function M. Write, "The volume of the dentin is maximized after approximately …"

C. Look at the graph and estimate at how many days the rate at which the volume of the dentin is growing is the fastest. This is asking you to look for the largest value of the rate of change of the function (i.e. the slope of M(t)). Write, "The volume of the dentin is growing at its fastest rate at approximately…."

D. Find the absolute maximum of M(t) on the interval [9, 51], using calculus, showing your work carefully. Compare your answer with the estimate you made from the graph in part B.

E. Find the absolute maximum of the rate of change of the volume of the dentin on the interval [9,51]. Here's how:

1. The rate of change of the volume of the dentin is M'(t), so begin by finding M'(t).

2. Now you want to find the maximum value of M'(t). When you find the maximum of a function $f(x)$, you take its derivative find its critical numbers, and calculate the values of $f(x)$ at each critical number and at each end point. The largest such value is the maximum value of f(x). Here you need to do that for M'(t). Use the notation M''(t) for the derivative of M'(t).