Homework Section 3.1 (Limits at Infinity)

I. The limits at infinity of rational functions (a rational function is a polynomial divided by a polynomial) can be calculated by following the instructions in the blue box on page 147. There is a pattern to what answers you can get. This exercise asks you about that pattern.

A. Do the WeBWork problem set LimitsAtInfinity.

B. What is the limit of the rational function shown below in each of the following cases? The coefficients (the *a*'s and *b*'s) be part of your answer.

$$\lim_{x \to \infty} \frac{a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x^1 + a_0}{b_m x^m + b_{m-1} x^{m-1} + \ldots + b_1 x^1 + b_0}$$

1. $m<n$.
2. $n<m$.
3. $m=n$.

II. Drug Concentration.

A. Do problem #79 page 153. Show your work; see Example 9 page 146 for an example. Notice in particular where the symbol "lim" appears, and where it does not, so that you can use the notation correctly in your homework.

B. Suppose the concentration of another drug in a patient's bloodstream h hours after it was injected is given by $B(h) = \frac{0.17h}{h^5 + 2}$. Calculate the limit of $B(h)$ as h tends to infinity.

C. Use Excel to compare the rates at which these drugs leave the system. After you have created the worksheet and the chart, as described below, answer the following questions (in sentences):

1. After how many hours is the concentration of each drug less than 0.01? (read the answer from the graph)
2. Which drug leaves the bloodstream the soonest, and how can you tell?

- Type "Time (hours)" in cell A1. Type "Concentration of Drug A" in cell B1 and "Concentration of Drug B" in cell C1. Adjust the column widths to fit the words (To adjust the width of column A, put your curser over the line between the A and the B at the top of the first two columns. Your curser will look like a vertical line with arrows pointing to the left and right. Click and drag the border between the two columns to the desired width.)
- Type 0 into cell A2 and 1 into cell A3. Select both cells A2 and A3 by clicking on A2 and dragging your curser to cell A3. Put your curser on the bottom right hand side of cell A3. Your curser should change into a small black plus shape. Hold the mouse so that the curser shows that plus shape, press the left mouse button, and holding it down, drag to the left to cell A22. You should now see the numbers 1 through 20 in these cells.
- In cell B2, you will type the formula for A(h), replacing h by A2. In cell B2, type "=(0.17*A2)/(A2^2+2)" and hit enter. Excel will calculate the value of A(h) when

h=0; the result should be the number 0 appearing in cell B2. When you click on cell B2 again, you can see the formula in the Formula Bar.

- In cell C2, type the formula for B(h), again replacing h by A2. It should look like =(0.17*A2)/(A2^5+2).
- Select cells B2 and C2 (by clicking on B2 and dragging to C2). Fill the formulas down to row 22, as follows. Put your curser on the bottom right hand side of cell C2. Your curser should change into a small black plus shape. Hold the mouse so that the curser shows that plus shape, press the left mouse button, and holding it down, drag to the left to cell A22.
- Save your work.

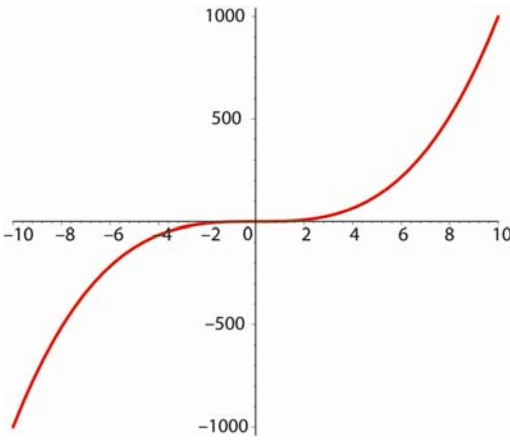
You will now graph these three functions.

- Select cells A1 through C22. Note that you are selecting the column headings as well as the data.
- Choose Chart... from the Insert menu. A dialog box called "Chart Wizard – Step 1 of 4 – Chart Type" will appear. Click on x-y scatter plot. There are five chart subtypes shown. Click on the second one (with the points and the curve showing). Click Next.
- The title of the dialog box changes to Step 2 of 4 – Chart Source Data. We won't use these options at this time. Click Next.
- Step 3 of 4 – Chart Options. Click on the Titles tab, if it is not already selected. Enter a title for your chart, and labels for the x and y axes. Click the tab at the top of the dialog box that says Gridlines. Check Major gridlines under Value (X) axis and Value (Y) axis. Click the tab at the top of the dialog box that says Legend. Click the circle for Bottom placement of the legend. Click Next.
- Step 4 of 4 – Chart Location. This dialog box allows you to choose whether the chart will appear in a new window, or as a part of the worksheet containing the data. Mark the "As an object in" radio button, if it is not already marked. Click Finish.

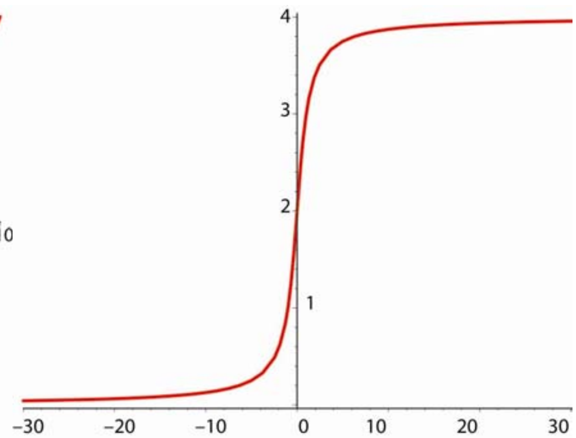
Print your worksheet along with the graph to include with your homework. Don't forget to answer the questions at the beginning of part C.

D. Suppose the concentration of a third drug is governed by the formula $C(h) = \dfrac{0.17h}{h^m + 2}$, where m is a positive number. Give an example of a value of m that would make drug C leave the system even sooner than drugs A and B? Explain why your example works.

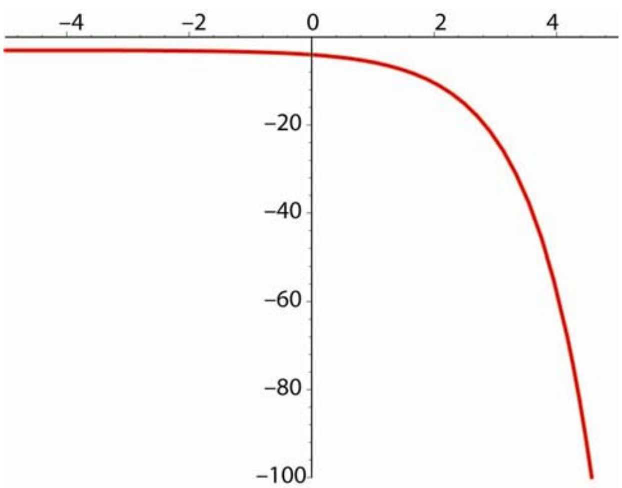
III. Calculate $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$ for each of the functions whose graphs are shown below. Your answer should be a number or $\pm\infty$.



Graph 1



Graph 2



Graph 3

IV. The probability of a population going extinct by time t can be estimated by

$$p(t) = \left(\frac{a[e^{(b-a)t} - 1]}{be^{(b-a)t} - a} \right)^N$$

, where a is the death rate, b is the birth rate (and $a \neq b$), and N is the number of individuals in the population at time $t=0$.

A. A population begins with 100 individuals. The birth rate is 25% per year, which means that in a given year, the number of births is 25% of the population at that time. The death rate is 30% per year, which means that in a given year the number of deaths is 30% of the population at that time.

1. Write the formula for $p(t)$ for this case. Your answer should have numerical values substituted in for a, b and N, so the only remaining variable is t.
2. Calculate the probability that this population will be extinct after 10 years, 50 years, 100 years and 150 years. Express each answer as a percent chance of extinction after the given number of years.
3. What does p(2) represent in words?
4. Calculate the limit as t tends to infinity of $p(t)$ in this case. Interpret the result (i.e. explain in words what this means for the population).

B. In part (A), the death rate was greater than the birth rate. Suppose you have a population with death rate, *a,* that is greater than the birth rate, *b*. This means that the term *b-a* in the formula is negative.

1. Calculate $\lim_{t \to \infty} e^{(b-a)t}$.
2. Calculate

$$\lim_{t \to \infty} p(t) = \lim_{t \to \infty} \left(\frac{a[e^{(b-a)t} - 1]}{be^{(b-a)t} - a} \right)^N .$$

   Explain your work.