Newberger, Calculus 119A, Fall 2006

Worksheet: areas

I. Use the graphs below to answer the following questions.

A. Just by looking at the graphs, explain how you can tell that the integrals $\int_{-4}^{6} f(x)\,dx$ and $\int_{-2}^{6} g(x)\,dx$ are both negative.

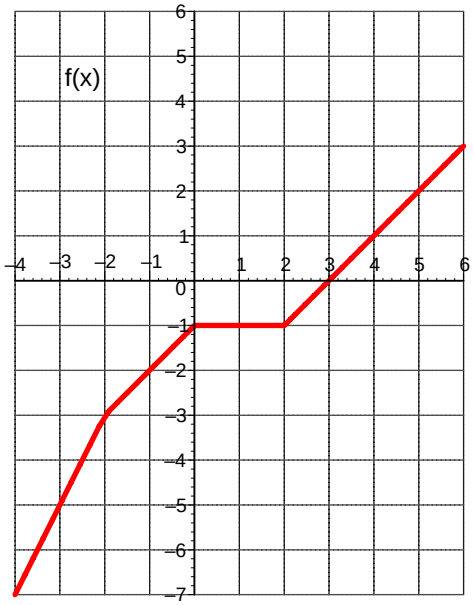
B. Find $\int_{-2}^{5} f(x)\,dx$ and $\int_{-2}^{6} g(x)\,dx$.

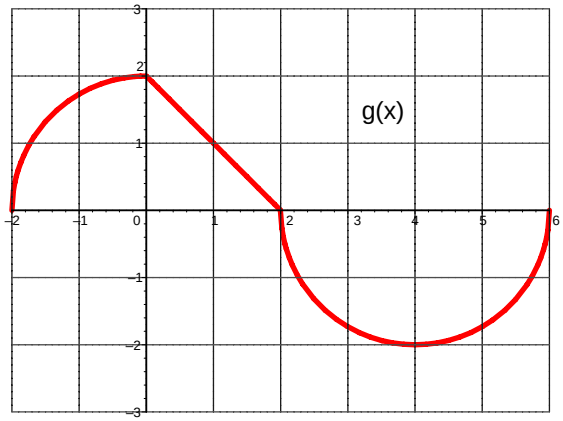
C. Find $\int_{2}^{4} f(x)\,dx$. How can you tell without calculating?

D. Which number is higher?

A. $\int_{3}^{4} f(x)\,dx$ or $\int_{3}^{5} f(x)\,dx$.

B. $\int_{2}^{3} g(x)\,dx$ or $\int_{3}^{4} g(x)\,dx$.

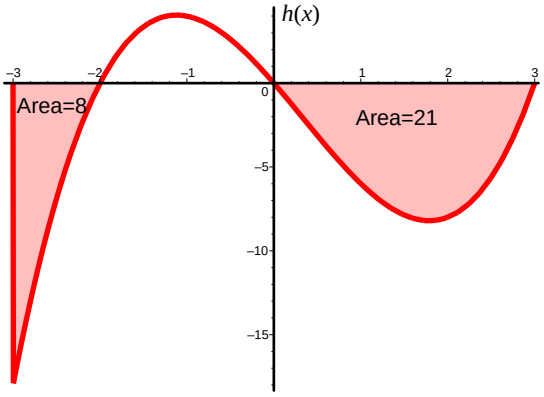




II. Use the graph below to answer the following questions.

A. Just by looking at the graph, determine whether $\int_{-2}^{0} h(x)\,dx$ positive or negative.

B. Find $\int_{-2}^{0} h(x)\,dx$, given that $\int_{-3}^{3} h(x)\,dx = -22$.



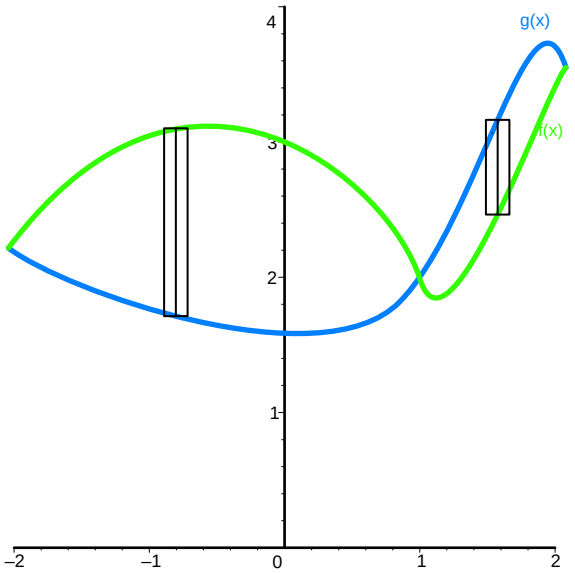
III. For each of the functions given below, graph $f(x)$ on the interval $[a, b]$ and find $\int_a^b f(x)dx$.

(a) $f(x) = -\sqrt{4 - x^2}$, $a = -2$ and $b = 2$.

(b) $f(x) = 1 + \sqrt{9 - x^2}$, $a = 0$ and $b = 3$.

IV. A. What are the heights of the approximating rectangles shown in the diagram below in terms of $f(x)$ and $g(x)$?

B. Write the area bounded by these two graphs as the sum of two integrals, again in terms of $f(x)$ and $g(x)$.



V. For each of the following, you will draw a graph of the region specified, and calculate its area, using the following steps:

- In these problems, you are given two functions $y = f(x)$ and $y = g(x)$, and two vertical lines $x = a$ and $x = b$.
- Draw coordinate axes for your graph showing the interval between $x = a$ and $x = b$, making it large enough to read easily. Draw in the vertical lines.
- Find the x- and y-intercepts for the functions $y = f(x)$ and $y = g(x)$.
- Find the x-coordinate at which the graphs of the functions intersect each other (if there are any) by setting them equal to each other and solving for x.
- Graph the functions $y = f(x)$ and $y = g(x)$ carefully and shade in the region.
- Write the area of the region as a sum of integrals. (You can draw the approximating rectangles if it makes it easier for you to figure out what function to integrate.)
- Calculate the integrals, either using area formulas (if you have triangles, rectangles or circles), or using antiderivatives.

A. The region between the graphs of $y = x + 1$ and $y = 1$ and between the vertical lines $x = -3$ and $x = 1$.

B. The region between the graphs of $y = 2x$ and $y = x^2 - 4$, and between the vertical lines $x = -2$ and $x = 1$. Tip: To graph $y = x^2 - 4$, begin by finding its vertex (the parabola $f(x) = ax^2 + bx + c$ has vertex $(\frac{-b}{2a}, f(\frac{-b}{2a}))$, and its x-intercepts.

C. The region between the graphs of $y = e^x$ and $y = 3 - e^x$, and between the vertical lines $x = -1$ and $x = 1$. Tip: The graph of $3 - e^x$ looks like the graph of e^x reflected across the x-axis (because of the $-$), and shifted up by 3.