Math 119A Fall 2006
Newberger

Exam 3 Review Sheet

This exam will cover Sections 5.1, 5.2, 5.3, 6.1, 6.3, 6.4, and 7.1. Practice on the problems given here. The problems on the exam will be similar to these, though not identical.

A. **Critical Numbers (Section 5.1)** You will be asked to find the critical numbers of a function, both from a graph and from a formula.

1. Let $f(x)$ be the function whose graph is given in exercise #7 page 275. Find the critical numbers of f and say at which critical numbers $f'(x) = 0$ and at which critical numbers $f'(x)$ does not exist.
   Answer: The critical numbers are $x = -7, -4$ and -2. $f'(x) = 0$ when $x = -7$ and when $x = -4$, and $f'(x)$ does not exist when $x = -2$.
2. Let $f(x) = x^2e^x$. Find the critical numbers of f and say at which critical numbers $f'(x) = 0$ and at which critical numbers $f'(x)$ does not exist.
   Answer: The critical numbers are $x = -2$ and 0. $f'(x) = 0$ when $x = -2$ and when $x = 0$, and $f'(x)$ always exists.
3. Let $f(x) = x^{4/3} - x^{1/3}$. Find the critical numbers of f and say at which critical numbers $f'(x) = 0$ and at which critical numbers $f'(x)$ does not exist.
   Answer: The critical numbers are $x = 0$ and $1/4$. $f'(x) = 0$ when $x = 1/4$, and $f'(x)$ does not exist when $x = 0$.
4. Let $f(x) = \dfrac{x-1}{x+1}$. Find the critical numbers of f and say at which critical numbers $f'(x) = 0$ and at which critical numbers $f'(x)$ does not exist.
   Answer: This function does not have any critical numbers. $f'(x)$ does not exist when $x = -1$, but neither does $f(x)$, so $x = -1$ is an asymptote not a critical number.

B. **Increasing and Decreasing Functions (Section 5.1)** You will be asked to find the intervals of increase and decrease for a function, both from a graph and from a formula. You may also be asked to draw a number line showing the intervals.

1. Let $f(x)$ be the function whose graph is given in exercise #7 page 275. Find the intervals on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing.
   Answer: $f(x)$ is increasing on the intervals $(-7, -4)$ and $(-2, +\infty)$, while $f(x)$ is decreasing on the intervals $(-\infty, -7)$ and $(-4, -2)$.
2. Let $f(x) = x^2e^x$. Find the intervals on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing.
   Answer: $f(x)$ is increasing on the intervals $(-\infty, -2)$ and $(0, +\infty)$, while $f(x)$ is decreasing on the intervals $(-2, 0)$.
3. Let $f(x) = x^{4/3} - x^{1/3}$. Find the intervals on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing.
   Tip: When you calculate the cube root of a negative number, like $(-2)^(1/3)$, your calculator may return an error. To get the result, calculate $(2)^(1/3)$ and then add the minus sign, (so $(-2)^(1/3) = -1.260$). When you calculate $(-2)^{4/3}$,

rewrite it as $((-2)^4)^(1/3)$, which is $(16)^(1/3)$ (which you can do on your calculator).

Answer: $f(x)$ is increasing on the intervals $(1/4, +\infty)$, while $f(x)$ is decreasing on the intervals $(-\infty, 0)$ and $(0, 1/4)$.

4. Let $f(x) = \dfrac{x-1}{x+1}$. Find the intervals on which $f(x)$ is increasing and the intervals on which $f(x)$ is decreasing.

Answer: $f(x)$ is increasing on the intervals $(infty, -1)$ and $(-1, +\infty)$. $f(x)$ is not decreasing on any intervals. (See the graph page 271).

C. **Local and Absolute Extrema (Sections 5.2 and 6.1)** You will be asked to determine the local and absolute maxima and minima of a function, and the values of x at which they are attained. Sometimes you will be asked to study a function only on a given interval.

1. Let $f(x)$ be the function whose graph is given in exercise #7 page 275. Find the local and absolute maxima and minima and the values of x at which they are attained.

Answer: $f(x)$ has an absolute minimum value of -2, which is attained at $x = -7$ and $x = -2$. The absolute minima are also local minima. $f(x)$ has a local maximum value of 3 attained at $x = -4$. $f(x)$ does not have an absolute maximum.

	x	$f(x)$	type of extrema
critical number	-7	-2	local and absolute minimum
critical number	-4	3	local maximum
critical number	-2	-2	local and absolute minimum

2. Let $f(x) = x^2e^x$. Find the local and absolute maxima and minima and the values of x at which they are attained on the interval [-1,1].

Answer: $f(x)$ has an absolute maximum value of 2.7183, which is attained at $x = 1$. The absolute maximum is also a local maximum. $f(x)$ has a absolute minimum value of 0, which is attained at $x = 0$. The absolute minimum is also a local minimum. $f(x)$ also has a local maximum value of 0.36788 attained at $x = -1$. We know $x = -1$ is a local maximum because $f(x)$ is decreasing to the right of $x = -1$ (using the increasing and decreasing intervals from part B).

	x	$f(x)$	type of extrema
end point	-1	0.36788	local maximum
critical number	0	0	absolute and local minimum
end point	1	2.7183	absolute and local maximum

3. Let $f(x) = x^{4/3} - x^{1/3}$. Find the local and absolute maxima and minima and the values of x at which they are attained on the interval [-1,1].

Tip: When you calculate the cube root of a negative number, like $(-2)^(1/3)$, your calculator may return an error. To get the result, calculate $(2)^(1/3)$ and then add the minus sign, (so $(-2)^(1/3) = -1.260$). When you calculate $(-2)^{4/3}$, rewrite it as $((-2)^4)^(1/3)$, which is $(16)^(1/3)$ (which you can do on your calculator).

Answer: $f(x)$ has an absolute maximum value of 2, which is attained at $x = -1$. The absolute maximum is also a local maximum. $f(x)$ has a absolute minimum value of -0.5725, which is attained at $x = 1/4$. The absolute minimum is also a local minimum. $f(x)$ also has a local maximum value of 0 attained at $x = 1$. We know $x = 0$ is neither a maximum nor a minimum because $f(x)$ is decreasing both to the left and the right of $x = 0$ (using the increasing and decreasing intervals from part B).

	x	$f(x)$	type of extrema
end point	-1	2	absolute and local maximum
critical number	0	0	neither a maximum nor a minimum
critical number	1/4	-0.4725	absolute and local minimum
end point	1	0	local maximum

4. Let $f(x) = \dfrac{x-1}{x+1}$. Find the local and absolute maxima and minima and the values of x at which they are attained.
   Answer: $f(x)$ has no critical numbers, and we are not given an interval, so we do not have any end points to study. So $f(x)$ has no local and no absolute extrema. It does have an asymptote at $x = -1$.

D. **Concavity and Inflection Points (Section 5.3)** You will be asked to find the intervals of concavity and the inflection points for a function, both from a graph and from a formula.

1. Let $f(x)$ be the function whose graph is given in exercise #7 page 275. Find the inflection points and intervals on which $f(x)$ is concave up and the intervals on which $f(x)$ is concave down.
   Answer: $f(x)$ has inflection points at (approximately) $x = -5.75$ and $x = -2$. $f(x)$ is concave down on the intervals $(-5.75, -2)$ and $(-2, +\infty)$, while $f(x)$ is concave up on the interval $(-\infty, -5.75)$.
2. Let $f(x) = x^2e^x$ Find the inflection points and the intervals on which $f(x)$ is concave up and the intervals on which $f(x)$ is concave down.
   Answer: $f(x)$ has inflection points at $x = -2 \pm \sqrt{20}$. $f(x)$ is concave down on the intervals $(-2 - \sqrt{20}, -2 + \sqrt{20})$, while $f(x)$ is concave up on the intervals $(-\infty, -2 - \sqrt{20})$ and $(-2 + \sqrt{20}, +\infty)$.
3. Let $f(x) = x^{4/3} - x^{1/3}$. Find the inflection points and the intervals on which $f(x)$ is concave up and the intervals on which $f(x)$ is concave down. Tip: When you calculate the cube root of a negative number, like $(-2)^(1/3)$, your calculator may return an error. To get the result, calculate $(2)^(1/3)$ and then add the minus sign, (so $(-2)^(1/3) = -1.260$). When you calculate $(-2)^{4/3}$, rewrite it as $((-2)^4)^(1/3)$, which is $(16)^(1/3)$ (which you can do on your calculator).
   Answer: $f(x)$ has an inflection point at $x = -1/2$ and $x = 0$. $f(x)$ is concave up on the intervals $(-\infty, -1/2)$ and $(0, +\infty)$, while $f(x)$ is concave down on the interval $(-1/2, 0)$.

4. Let $f(x) = \dfrac{x-1}{x+1}$. Find the inflection points and the intervals on which $f(x)$ is concave up and the intervals on which $f(x)$ is concave down.

   Answer: $f(x)$ does not have any inflection points. $f(x)$ is concave up on the intervals $(-\infty, -1)$, while $f(x)$ is concave down on the interval $(-1, +\infty)$.

E. **Implicit Differentiation (Section 6.3)** You will be asked to find y', the derivative of y with respect to x, given an equation that relates x to y. Practice on problems 1-16 in Section 6.3, on the problems from the in class activity on implicit differentiation, and on the WeBWork problems. You may also be asked to find the equation of the tangent line at a given point, as in problems 19-26 in Section 6.3.

F. **Related Rates Problems (Section 6.4)** You will be asked to find the rate of change of a function with respect to time in a word problem. See for example, problems 11-19 in Section 6.4. Make sure to include units on your answers.

G. **Antiderivatives (Section 7.1)** You will be asked to find the antiderivative of functions using the rules and formulas from Section 7.1. Practice on WeBWork and on problems 1-40 in Section 7.1. Note in particular problems like #15 and #23 in which you must rewrite the function before taking the antiderivative.