Topology II, Newberger, Fall 2005

Homework: Section 70. Due Wednesday, November 23.

Let X_n be the following circle in $\mathbb{R}^2$:

$$X_n = \{(\cos(2\pi x) - (2n-2), \sin(2\pi x)) | x \in [0,1]\}.$$

Consider $W_n = X_1 \cup \cdots \cup X_n$, under the subspace topology in $\mathbb{R}^2$.

1. Let $x_0 = -1 \times 0 \in W_2$. Find two subsets $U, V \subset W_2$, containing x_0, such that
   a. U and V are open in W_2.
   b. X_1 is a deformation retract of U, and X_2 is a deformation retract of V.
   c. $U \cap V$ is simply connected.

   Prove rigorously that (a), (b) and (c) hold for your sets U and V. Apply the Seifert-Van Kampen Theorem (in particular Corollary 70.3) to show that $\pi_1(W_2)$ is isomorphic to $\mathbb{Z} * \mathbb{Z}$, the free group on two generators.
2. Use induction on n to show that $\pi_1(W_n, x_0)$ is isomorphic to $\mathbb{Z} * \cdots * \mathbb{Z}$ (n-times). (Remember that $\pi_1(W_n, x_0)$ is isomorphic to $\pi_1(W_n, x_1)$ for any other $x_1 \in W_n$, so you can change your base point as necessary.)