

## Topology II, Newberger, Fall 2005

### Final Exam

- I. In this exercise, I expect you to prove the Seifert-Van Kampen Theorem(70.2). The proof is in the text, written in paragraphs, and with some details omitted. Your proof should be more like an outline, with each statement that you make justified clearly. Make sure to address the points listed in the instructions below.
  - A. The first claim in the proof is that  $j$  is surjective. Suppose that  $[f]_X \in \pi_1(X, x_0)$ . Explain what the word “generates” means in this context, what elements of  $\pi_1(U, x_0) * \pi_1(V, x_0)$  look like, and how we know there is a  $q \in \pi_1(U, x_0) * \pi_1(V, x_0)$  such that  $j(q) = [f]$ .
  - B. The second claim in the argument is that  $N \subset \ker(j)$ . Explain the proof of this statement, making sure to justify each step. In particular, the proof in the book includes a sequence of equalities. Justify each equality, and explain how that sequence of equalities implies the statement.
  - C. The third claim is that  $j$  induces an epimorphism  $k$ . Explain what the map  $k$  is, and why it exists.
  - D. The fourth claim is that to prove that  $\ker(j) = N$ , it suffices to show that  $k$  has a left inverse. Explain this claim.
  - E. Quote the statement of Theorem 70.1, and explain how it is used to complete the proof of this theorem.
- II.
  - A. Read the definition of an  $n$ -fold dunce cap on page 443. Write the  $n$ -fold dunce cap as a polygon with a labelling scheme.
  - B. Prove that the fundamental group of the dunce cap is the cyclic group of order  $n$ . See the proof of Theorem 34.4, or use Theorem 74.2.
- III. Do problem 3 on page 454.