Newberger, Math 122, Fall 2005
Homework: Sections 2.1 and 2.2 *Due: Tuesday, September 6*

(1) Read Example 2 page 89. I'm going to tell you what I see in that example, so that you can more easily answer the questions for the problem that you will do (#4 page 41). There is nothing to turn in for number (1).
Note that in the problem, you are given the following information:

Type of information	What was given
the input of the function (including units)	t was time in seconds after the flash goes off
the output of the function (including units)	$Q(t)$ is the charge after t seconds measured in microcoulombs
data	see table top of page 89 (input and output values)
a point P	here P is at $t = .04$, so $P = (.04, 67.03)$

With this information, they do the followinng:

(a) They graph the data. They do not label the axes of the graph with "time (seconds)" and "charge (microcoulombs)," *as they should have!*
(b) They plot P on the graph and draw the tangent line to the graph at P.
(c) They create a table showing the slope M_{PR} of the secant line connecting P to each of the points R given in the original table. Note that they show their work for the first computation, which was the slope of the secant line connecting $P = (.04, 67.03)$ to $R = (0.00, 100.00)$.
(d) They averag the two values of m_{PR} for the secant lines that lie on either side of the tangent line at P. (The answer was -674.75; do you see it? Note which points R they used to get this average.)
(e) They estimated the slope of the tangent line in a different way, by reading of the height (ΔQ) and the width (Δt) of the triangle whose hypotenuse is the tangent line at P and dividing. For this to be accurate, the drawing of the triangle must be quite good; I will not have you do this.
(f) In the margin under the second table, they write a sentence saying what the physical interpretation of the slope is, in terms of the original word problem. Q is charge, and t is time so

$$\frac{\Delta Q}{\Delta t} = \frac{\text{charge}}{\text{time}} = \text{current}.$$

Current is a word that actually means the rate or ratio of charge per unit time.
You can get the units for the slope in this fashion also: Q is in microcoulombs, and t is in seconds, so the units on the slope slope is

$$\frac{\Delta Q}{\Delta t} = \frac{\text{microcoulombs}}{\text{second}} = \text{microamperes}.$$

The definition of 1 microampere is 1 microcoulomb/second.

(2) Now you will do exercise #2 page 91, which is similar to Example 2 page 89, following my instructions below (most of the instructions are worth points; you'll lose points for not doing all of them). I hope what I wrote above will help you. Do your homework on another piece of paper. Clearly write the page and problem number, where I can see it.

(a) Copy this table onto your homework and fill it out. Try to include as much information as I included in my table above.

Type of information	What was given
the input of the function (including units)	
the output of the function (including units)	
data	Copy the data into your homework somewhere.
the point that the question is about (two coordinates)	

(b) Draw a graph that is large enough to read (maybe covering half a sheet of paper) displaying this data. Graph paper is nice, but not required. To deal with the large numbers, use a zig zag like they did when graphing the charge on page 89. Label your axes. Sketch in a curve connecting the points. Draw in the tangent line to the graph at the point P at $t = 42$ that we are studying in this problem.

(c) Do part (a) of the problem in the book. Here you are calculating the slope of the secant line connecting the point P at $t = 42$ to the point at R at $t = 36$. Show your work by writing a sentence and doing a computation similar to the one under Figure 4 on page 89.

(d) Fill out the table below with the rest of your answers. You do not need to show your work for the rest of the computations.

part	t at R	R	m_{PR}
(a)	36	(36,2530)	your answer = the slope of the secant line connecting P to R
(b)			
⋮			

(e) Average the slopes of the two secant lines for the points nearest to P to estimate the slope of the tangent line at P. Write a sentence saying what your answer represents in terms of the word problem (Hint: the physical interpretation of the slope is given in the problem, where it says "the slope of the tangent line represents..."

(3) Exercise #4 page 91 is a similar problem, but it does not have a story to go along with it. Answer the following questions; I have you doing some of the parts out of order, because it seems more logical that way. Use your calculator to get the values of the natural log. Express all of your answers with 6 decimal places.

(a) Draw a graph of the function $f(x) = \ln(x)$. Label the graph "$ln(x)$." You can copy the shape from the graph of $\log_a(x)$ (for $a > 1$), given on page 68. (This works because $\ln(x)$ is the same thing as log base e, i.e. $\log_e(x)$, where e is the irrational number $2.71\ldots$, which is greater than 1. Again make your graph large enough to read, maybe half a page.)

(b) For what values of x is $\ln(x) = 0$? (Read it off the graph if you don't know.) I always expect you to answer in sentences, or at least clearly indicate what the question was. Don't just write a single number. Here, you could write "$\ln(x) = 0$ when $x = ...$" On your diagram, label the point at which the graph crosses the x-axis.

(c) What is the domain of $f(x) = \ln(x)$?
Answer "The domain of $f(x) = \ln(x)$ is..."

(d) Add the point $P(2, \ln(2))$ to your graph. Sketch the tangent line to the graph at P.

(e) In this problem, Q is the point $(x, \ln(x))$. If x is 1.5, then what is Q? Express your answer with six decimal places. Find the slope of the secant line connecting P to Q for $x = 1.5$. Show your work carefully.

(f) Fill out the following table. No need to show your work.

part	x at Q	Q	m_{PQ}
(i)	1.5	$(1.5, \ln(1.5))$ (write the number in)	your answer = the slope of the secant line connecting P to Q
(ii)			
⋮			

(g) Using the results in your table, guess the value of the slopoe of the tangent line to the curve at $P(2, \ln(2))$. Write your answer in a sentence.

(4) (a) Read Section 2.2 from the beginning through the paragraph after Example 5 on page 97.

(b) Do problems #18 and #20 on page 103. Create tables to display your answers. Write your finals estimates by writing out the entire expressions; for example in #18, write

$$\lim_{x \to 0^+} x \ln(x + x^2) = ...$$