Newberger, Calculus 122, Fall 2005

Worksheet: Section 4.4, 4.5 (and 4.7) Graphing, L'Hopital's Rule (and More Optimization)

I. Graphing in 100 easy steps (just kidding, there are only 8 steps).

1. Consider the function $g(x) = \frac{x}{x^2-9}$.
   A. Find the domain of g.
   B. Find the x- and y-intercepts of g.
   C. Is g an even function, an odd function, or a periodic function (or none of the above)?
   An even function satisfies $f(-x) = f(x)$. Its graph is symmetric across the y-axis.
   An odd function satisfies $f(-x) = -f(x)$. Its graph on the positive x-axis looks like an upsidedown version of its graph on the negative x-axis.
   A periodic function satisfies $f(x + c) = f(x)$ for some positive number c. Its graph repeats itself on every interval of length c.
   Upshot: Calculate $g(-x)$ to see if you get $-g(x)$ or $g(x)$. This will determine if your graph is either odd or even (it may be neither). In this section, only the trigonometric functions will be periodic.
   D. Find the vertical and horizontal asymptotes of g, if any.
      i. Horizontal asymptotes. Find the limits as $x \to \pm\infty$ to determine the horizontal asymptotes, if any. Use L'Hospital's rule, if it applies.
      ii. Vertical asymptotes. If g has a vertical asymptote at $x = a$, find the limits as $x \to a^+$ and as $x \to a^-$.
   E. Find the intervals on which g is increasing or decreasing. Include the asymptotes and the critical numbers on your number line, when finding the end points of your intervals.
   F. Find the local maxima and minima of g. The local maxima and minima will not occur at asymptotes, since local maxima and minima must be in the domain of the function.
   G. Find the intervals on which g is concave up or concave down. Include the asymptotes as endpoints for your intervals. Find the inflection points of g; asymptotes do not count as inflection points, since inflection points must be in the domain of the function.
   H. Graph g.
2. Repeat the above exercise for the function $h(x) = e^x/x$.

II. Tricky L'Hospital's Rule problems. For each of the following problems, (a) determine which indeterminant form you have, (b) manipulate it into either $\frac{0}{0}$ or $\frac{\infty}{\infty}$, and (c) use L'Hospital's rule.

1. $\lim_{x\to\infty} x^3 e^x$

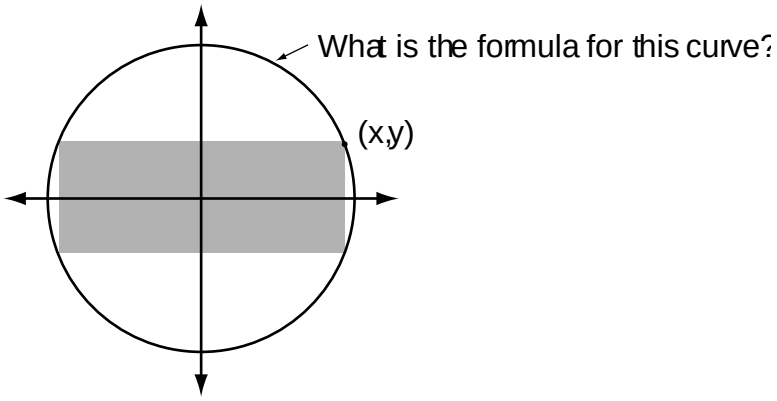
2. $\lim_{x\to 0^+} x\ln(x)$

3. $\lim_{x\to 0^+} \sin x \ln(x)$

4. $\lim_{x\to 0^+} x^{x^2}$

5. $\lim_{x\to 0^+} (\cos x)^{1/x^2}$

III. 1. Find the dimensions of the rectangle of largest area that can be inscribed inside a circle of radius r.



2. Find the dimensions of the rectangle of largest area that can be inscribed inside an equilateral triangle of side length L.