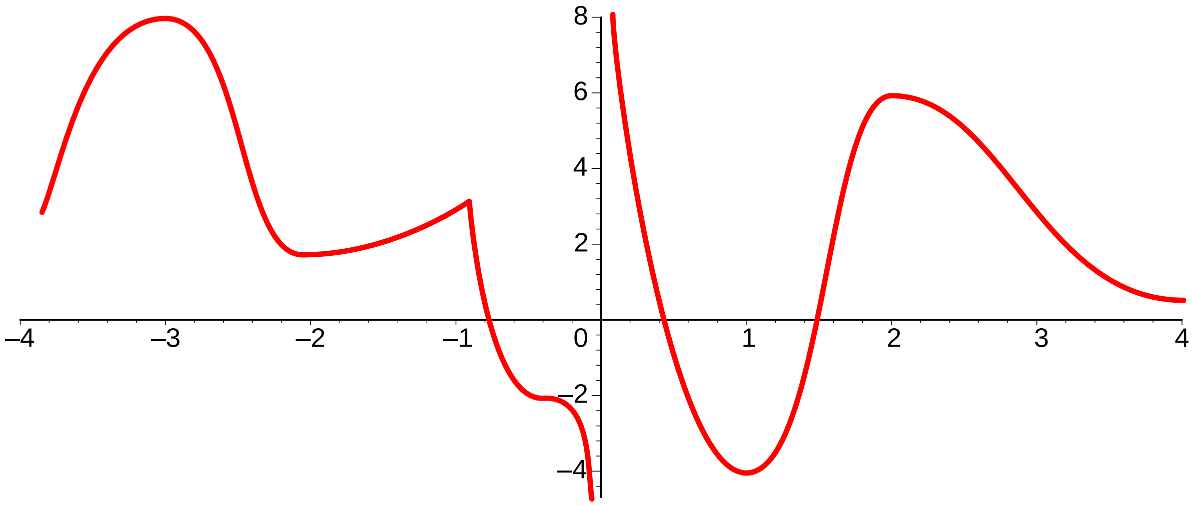
Newberger, Calculus 122, Fall 2005

Worksheet: Section 4.1, 4.3 and 4.7 Extreme values and Optimization

I. The following questions refer to this graph of the function of f.



a. What are the critical numbers of f? Remember, critical numbers are the x-values in the domain of f at which $f'(x) = 0$ or $f'(x)$ does not exist.

b. Does f have a global max or global min?

c. What are the local maximum and minimum values of f and where are they attained?

d. On what intervals is $f'(x)$ positive?

e. On what intervals is $f''(x)$ positive?

f. At which values of x does the concavity change from up to down or from down to up?

g. Which values of x are inflection points? Remember, an inflection point is a point at which the concavity changes from up to down or from down to up and at which f is continuous.

II. 1. Consider the function $f(t) = t\sqrt{4 - t^2}$ on the interval $[-1, 2]$. Do the following on the back of the worksheet.

a. Find the critical numbers of f. Which of these lie in the interval in which we are interested?

b. Make a table with the end points and the critical numbers that lie in our interval in the first column and the values of f at these points in the second column.

c. At what values of x does f attain its global maximum on $[-1, 2]$? What is its maximum value?

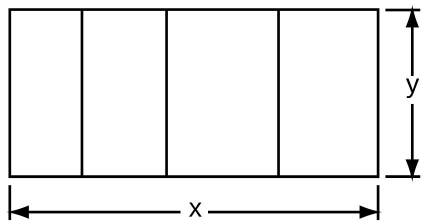
d. At what value of x does f attain its global minimum on $[-1, 2]$? What is its minimum value?

2. Consider the function $f(x) = xe^{-x}$ on the interval $[0, 2]$. Repeat the exercise above to find the global maximum and minimum values of f on this interval, and the x values at which they are attained.

III. Consider the function $g(x) = x^4 - 6x^2$. Notice that this function is a polynomial, so it is continuous for all real numbers. This type of problem can be more complicated if the function you are studying has vertical asymptotes (i.e. is not continuous everywhere). We will consider such functions next week. Do your work on another piece of paper.

1. The First derivative test is for finding the intervals on which g is increasing or decreasing.
   a. Find $g'(x)$.
   b. Find the critical numbers of g.
   c. plot the critical numbers of g on a number line, and choose test points within each interval.
   d. Determine the sign of $g'(x)$ on each interval, using your test points, and mark it on your number line.
   e. On what intervals is $g(x)$ increasing? On what intervals is $g(x)$ decreasing?
   f. At what values of x are the local maxima and minima attained?
2. The Second Derivative Test is a short cut for determining the local maxima and minima of the function without discussing the intervals on which g is increasing and decreasing.
   a. Find the critical numbers at which g is continuous.
   b. Find $g''(x)$.
   c. Make a table showing the critical numbers from (a) in the first column and the values of $g''(x)$ at each of these points in the second column. In the third column indicate if the critical number is a local maximum or a local minimum, using the second derivative. Your answer should agree with the previous part.
3. Now you will find the intervals on which g is concave up and concave down, and the inflection points.
   a. Find the values of x at which $g''(x) = 0$ or $g''(x)$ does not exist. You don't have them in this problem, but if you did, you would include the points at which g and g' do not exist.
   b. Plot them on a number line, and choose test points.
   c. Determine the sign of g'' on each of the intervals.
   d. On what intervals is g concave up, and on which intervals is g concave down?
   e. What are the inflection points of g?
4. Now you will graph your function, using the information that you worked out above. In parts (a) and (b) below, you will find a little more information that will make your graph more accurate (using (a)) and easier to draw (using (b)).
   a. Find the zeros of g. I.e. set $g(x) = 0$ and solve for x.
   b. Find the limits $\lim_{x\to+\infty} g(x)$ and $\lim_{x\to-\infty} g(x)$.
   c. Make a table with the following columns. Label the first column "type of point;" in this column, you will fill in local max, local min, inflection point or zero (from part a). Label the second column x-value; in this column, you will list all the local maxima and minima, zeros and inflection points of g. Label the third column $g(x)$; in this column, evaluate g at each of the numbers in the previous column.
   d. Plot the points in your table on a graph. Sketch the graph of the function g, connecting the dots. Make sure your graph also satisfies the limits you calculated in part (b).

IV. 1. A farmer with 750 ft of fencing wants to enclose a rectangular area and then divide it into four pens with fencing parallel to one side of the rectangle. What is the largest possible total area of the four pens?



(i) Typically, you would begin by drawing the picture and choosing your variables; I have done this for you in this problem.
(ii) What do you want to optimize (i.e. what to you want to find the maximum or minimum of)?
(iii) Write a formula for the total area of the pen.
(iv) You will have more than one variable in your formula, so you will need to use the information given to eliminate one of them (we only really know how to find maxima and minima for functions of one variable. Here you are given that the farmer has 750 ft of fencing. Use that to write a formula with your variables. Solve it for one of the variables (it doesn't matter which variable you solve for here, but lets do x so we all have the same thing), and use substitution to write your formula for total area in terms of x only.
(v) Find the intervals on which the area is increasing and decreasing by plotting them on a number line. Use this information to find the global maximum of the area function.

2. If 1200 cm^2 of material is available to make a cardboard box with a square base and an open top, find the largest possible volume of the box.
(i) Draw a picture and choose your variables.
(ii) What do you want to optimize? Write a formula for it, using your variables.
(iii) Use the given information to reduce your formula to one variable. (Solve for the variable that is easiest to solve for.) The area of the cardboard used in your box is the sum of the areas of the sides plus the bottom.
(iv) Find the intervals on which the volume of the box is increasing and decreasing by plotting them on a number line. Use this to determine the absolute maximum volume of the box.