Newberger, Calculus 122, Fall 2005

Worksheet: Section 3.5, 3.6 and 3.10 The Chain Rule and Implicit Differentiation

I. Each of the functions below is the composition of two functions $h(x) = f(g(x))$. Think of f as the outside function and g as the inside function. Identify f and g and then use the chain rule to take the derivative.

1. $h(x) = \sin(4x)$
   $f(x) =$____________
   $g(x) =$____________
   $h'(x) =$____________

2. $h(x) = \sqrt{x^2 + 5}$
   $f(x) =$____________
   $g(x) =$____________
   $h'(x) =$____________

II. Think of the chain rule as "The derivative of the outside function evaluated at the inside function times the derivative of the inside function." Use this mnemonic to help you apply the chain rule without writing out the f and g as you did in part I.

1. $h(x) = \sin(x^3)$
2. $h(x) = \tan(\cos(x))$
3. $h(x) = (x^3 + 4x)^7$
4. $h(x) = \sin^7(x)$
5. $h(x) = \sqrt[4]{x^6 + 4x^2}$
6. $h(x) = (x^3 + 5)^{2/3}$
7. $h(x) = e^{4x^2}$
8. $h(x) = e^{\sin(x)}$

III. These problems are compositions of more than two functions. Use the chain rule (repeatedly) to find the derivatives.

1. $h(x) = 6\tan(x^3)$
2. $h(x) = \sqrt{\sin(8x^2)}$
3. $h(x) = \sin^2(3x^7)$
4. $h(x) = \sqrt{1 + \sqrt{1 + x^4}}$

IV. These problems combine the chain rule with the product and quotient rule.

1. $h(x) = (x^4 + 1)^3(3 + x^2)$
2. $h(x) = xe^{-x^2}$
3. $h(x) = \sqrt{\dfrac{x+1}{x-1}}$
4. $h(x) = \dfrac{e^{2x}}{1 + e^{4x^3}}$

V. For the following problems, think of y as a function of x, so every time you differentiate y, you will get $\frac{dy}{dx}$. Differentiate the following.

1. y^5

2. $x^2 + y^4$

3. xy

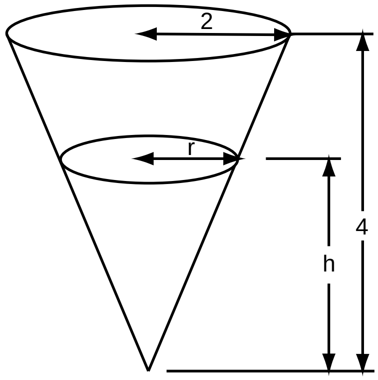
4. $\dfrac{y}{x}$

VI. Use implicit differentiation to differentiate both sides of the given equations with respect to x. Then solve for $\frac{dy}{dx}$.

1. $x^3 + x^2y + 4y^2 = 6$

2. $4\cos(x)\sin(y) = 1$

A water tank has the shape of an inverted circular cone with base radius 2m and height 4m. If water is being pumped into the tank at a rate of $4\text{m}^3/\text{min}$, find the rate at which the height of the water is rising when the water is three meters deep.



Variables:
h = the height of the water at a given time. Units: m
r = the radius of the top of the water at a given time. Units: m
V = the volume of water at a given time. Units: m^3
t = time. Units: sec
dh/dt = the rate at which the height of the water is rising at a given time. Units: m/sec
dV/dt = the rate at which the volume of water is changing at a given time. Units: m^3/sec

These quantities are given:
Height of the tank=__________________________
Radius of the tank=__________________________
dV/dt =__________________________

What does the question ask for? __________________________

a. The volume of a cone is $V = \frac{1}{3}\pi r^2 h$. You want to find the rate of change of the height of the water, so you want to make this into a function of height alone (getting rid of the r). Use similar triangles to get an equation relating r and h.

b. Solve your relationship for r and substitute it into the equation for volume, making the volume into a function of h alone.

V =__________________________

c. Use implicit differentiation to differentiate both sides of the equation with respect to time. Don't forget your dV/dt and dh/dt.

d. Since the question asks how fast the water is rising when the height is 3m, substitute in $h = 3$, and the given value of dV/dt. Solve for dh/dt. Express your final answer in a sentence.