Newberger, Calculus 122, Fall 2005

Worksheet: Section 3.1, 3.2 and 3.4 Derivatives

I. The derivatives of $\sin x$ and e^x from their graphs.

A. Sine. Let $f(x) = \sin(x)$.

1. On the back of this worksheet, draw the graph of $f(x) = \sin(x)$ as x ranges at least from -2π to 4π. Make your drawing extend all the way across the page, so you can see what you are doing. Draw a set of axes directly below your graph, on which you will graph the derivative $f'(x)$ after you answer the following questions.
2. At what values of x is the slope of the tangent line to the graph of $\sin(x)$ equal to zero? In other words, for what values of x is $f'(x) = 0$? ____________
3. On your CPR assignment, you found that the slope of the tangent line to the graph of $\sin(x)$ at the point $x = 0$ is 1. Using symmetry, at what values of x is the slope of the tangent line to the graph of $\sin(x)$ equal to 1? In other words, for what values of x is $f'(x) = 1$? ____________
   At what values of x is the slope of the tangent line to the graph of $\sin(x)$ equal to -1?____________
4. Plot the points at which $f'(x)$ is -1, 0 and 1 on your second axes. Complete your sketch the graph of $f'(x)$.
5. The graph of $f'(x)$ should look like a familiar function. What function is it?
6. Check your answers. Upshot:____________

B. Exponential function. Let $g(x) = e^x$.

1. The graphs of exponential functions like 2^x, 3^x, $(1/2)^x$ and e^x all pass through the point $(0, 1)$ on the y-axis. However they have different slopes at that point. Use a calculator to find the value of the irrational number e, by putting in e^1.
   $e =$____________
   This number e is special because, unlike 2^x, 3^x, and all the other exponential functions, the tangent line to the graph of the exponential function e^x has slope 1 at the point $x = 0$.
2. Sketch the graph of e^x on the back of this worksheet, making the slope of the tangent line to the graph at $x = 0$ equal to 1.
3. Directly below your graph of $g(x) = e^x$, sketch the graph of the derivative $g'(x)$.
4. The graph of $g'(x)$ should look like a familiar function. What function is it?
5. Check your answers. Upshot:____________

II. Using formulas: the power rule. Calculate the following derivatives.

(a) $\frac{d}{dx}x^5 =$

(b) $\frac{d}{dx}(7x^5 + 3x^2 + 2x + 8) =$

(c) $\frac{d}{dx}x^{-4} =$

(d) $\frac{d}{dx}\left(\frac{5}{x^2}\right) =$

(e) $\frac{d}{dx}x^{2/3} =$

(f) $\frac{d}{dx}\sqrt[5]{x^2} =$

Simplify before calculating.

(g) $\frac{d}{dx}x^5(4x^7 - 3x) =$

(h) $\frac{d}{dx}\left(\frac{\sqrt[3]{x^4}+x^2}{x^3}\right) =$

Check your answers.

III. Using formulas: the derivatives of $\sin(x)$, $\cos(x)$ and e^x. Calculate the following derivatives.

(a) $\frac{d}{dx} 5\cos(x) =$

(b) $\frac{d}{dx} \sin(x) - \cos(x) =$

(c) $\frac{d}{dx} \pi e^x =$

(d) $\frac{d}{dx} (e^{x+3}) =$

IV. Product rule.

A. Consider the function $k(x) = x^2 \sin(x)$.

1. $k(x)$ is a product of two functions. In other words, $k(x) = f(x)g(x)$, where $f(x) =$____________________ and $g(x) =$____________________
2. Calculate the derivatives.
   $f'(x) =$ ____
   $g'(x) =$____
3. Use the product rule to find $k'(x)$.

B. Consider the function $m(t) = \dfrac{8t^2 + t^3 + 9}{3t^7 - t^{2/3}}$.

1. $m(t)$ is a quotient of two functions. In other words, $m(t) = f(t)/g(t)$, where $f(t) =$____________________ and $g(t) =$____________________
2. Calculate the derivatives.
   $f'(t) =$____
   $g'(t) =$____
3. Use the quotient rule to find $m'(t)$.

C. Consider the function $r(x) = x^{2/3}\cos(x)$. When you read the product rule, you can read it this way: "The first function times the derivative of the second, plus the second function times the derivative of the first." Use this mnemonic to calculate $r'(x)$ using the product rule, without writing out f and g as you did above.

D. Consider the function $f(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$. When you read the quotient rule, you can read it this way: "The bottom times the derivative of the top, minus the top times the derivative of the bottom, all over the bottom squared." Use this mnemonic to calculate $f'(\theta)$ using the product rule, without writing out f and g as you did above.

Upshot: $\frac{d}{dx} \tan(x) =$____________________