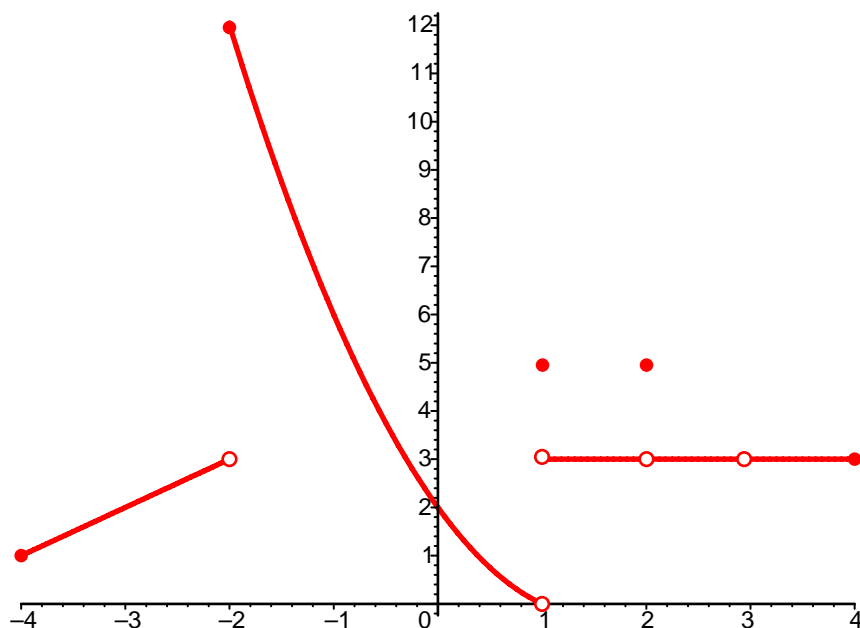


**Worksheet: Limits of piecewise functions**



Determine each of the following from the graph of  $f$  shown here.

$\lim_{x \rightarrow 0} f(x) =$ _____	$\lim_{x \rightarrow 2} f(x) =$ _____
$\lim_{x \rightarrow -2^-} f(x) =$ _____	$\lim_{x \rightarrow -2^+} f(x) =$ _____
$\lim_{x \rightarrow 3} f(x) =$ _____	$\lim_{x \rightarrow 1} f(x) =$ _____
$f(-2) =$ _____	$f(2) =$ _____

Using the graph above, find all values of  $a$ , with  $-4 \leq a \leq 4$ , having each of the following properties.

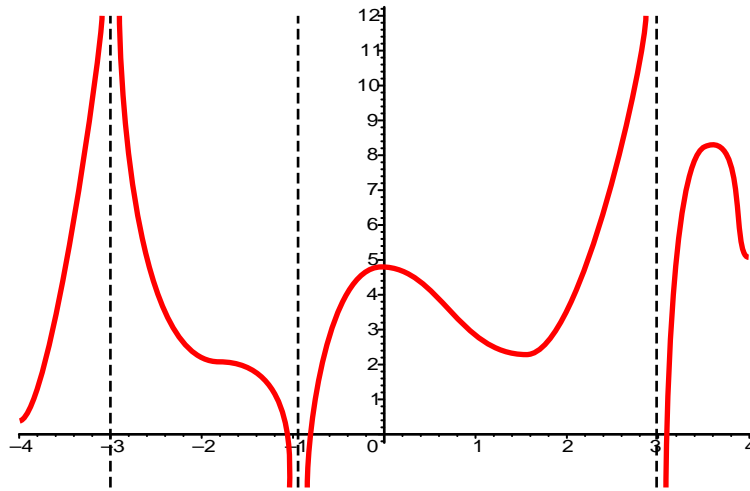
- $f(a)$  does not exist. \_\_\_\_\_
- $\lim_{x \rightarrow a} f(x)$  does not exist, but  $a$  is in the domain of  $f$ . \_\_\_\_\_
- $\lim_{x \rightarrow a} f(x)$  exists, and  $a$  is in the domain of  $f$ , but  $\lim_{x \rightarrow a} f(x) \neq f(a)$ . \_\_\_\_\_

On the back of this worksheet, draw a graph of a function  $h(x)$  satisfying all of the following requirements.

$\lim_{x \rightarrow -2} h(x) = 2$	$h(-2) = 4$
$\lim_{x \rightarrow 0^-} h(x) = 1$	$\lim_{x \rightarrow 0^+} h(x) = -1$
$\lim_{x \rightarrow 1^-} h(x) = 0$	$\lim_{x \rightarrow 1^+} h(x) = 2$
$h(1) = 2$	$h(0) = 3$

For the function  $g$  given here, find  $\lim_{x \rightarrow -1} g(x)$  and  $\lim_{x \rightarrow 1} g(x)$  if they exist.

$$g(x) = \begin{cases} x^2 + 1 & \text{if } x < -1 \\ 2x + 4 & \text{if } -1 \leq x \leq 1 \\ 2 - x & \text{if } 1 < x \end{cases}$$



Determine each of the following from the graph of  $f$  shown here.

$$\lim_{x \rightarrow -3^-} f(x) = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow -1} f(x) = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow 3^-} f(x) = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow 3^+} f(x) = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow 3} f(x) = \underline{\hspace{2cm}}$$

Formulas for the vertical asymptotes:

Consider the function

$$h(x) = \frac{x^2 - 4}{(x + 1)(x + 3)^2}.$$

To determine  $\lim_{x \rightarrow -1^-} h(x)$ , first try plugging  $x = -1$  into  $h(x)$ .

Calculating  $h(-1)$  yields  $\underline{\hspace{2cm}}$ .

Anytime you get (any nonzero number)/0, then your limit will either be  $+\infty$  or  $-\infty$ , and you have to do more work to determine which.

Mark the  $x$ -values at which  $h$  is zero or undefined on a number line. ( $h$  is zero where the numerator is zero, and  $h$  is undefined where the denominator is zero.)

Which interval of  $x$ -values do we need to study in order to determine  $\lim_{x \rightarrow -1^-} h(x)$ ?

(Circle positive or negative in these sentences.) On the interval you just named, the numerator of  $h$  is positive/negative, and the denominator is positive/negative. So the function  $h$  is positive/negative on that interval. Thus

$$\lim_{x \rightarrow -1^-} \frac{x^2 - 4}{(x + 1)(x + 3)^2} = \underline{\hspace{2cm}}$$

Use similar arguments to determine the following; show your work on the back:

$$\lim_{x \rightarrow -1^+} h(x) = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow -3^+} h(x) = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow -3^-} h(x) = \underline{\hspace{2cm}}$$