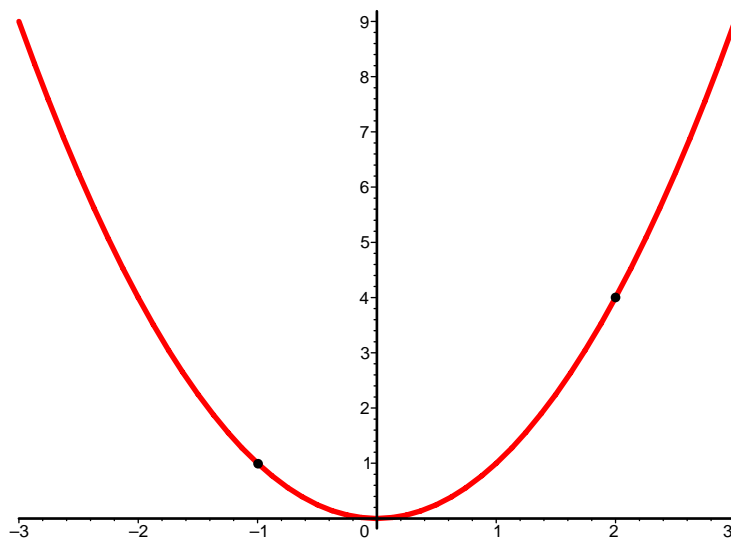


Worksheet: Calculating the slope of a tangent line



The graph of $f(x) = x^2$

- (1) What is $f(2)$?
- (2) What are the coordinates of the point on the graph above the number 2 on the x -axis? Label that point on the graph with its coordinates.
- (3) What is $f(-1)$?
- (4) What are the coordinates of the point on the graph above the number -1 on the x -axis? Label that point on the graph with its coordinates.
- (5) Draw the secant line connecting these two points. What is the slope of this secant line?

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \underline{\hspace{2cm}},$$

with $(x_1, y_1) = \underline{\hspace{2cm}}$, and $(x_2, y_2) = \underline{\hspace{2cm}}$.

Now do the same exercise with a variable x instead of -1 .

- (1) What is $f(x)$?
- (2) What are the coordinates of the point on the graph above the number x on the x -axis?
- (3) Draw the secant line connecting these two points. What is the slope of this secant line?

$$s(x) = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \underline{\hspace{2cm}},$$

with $(x_1, y_1) = \underline{\hspace{2cm}}$, and $(x_2, y_2) = \underline{\hspace{2cm}}$.

- (4) You have just created the function $s(x)$ whose output is the slope of the secant line connecting the point $(2, 4)$ to the point on the graph above the number x on the x -axis. What does $s(3)$ represent? Write your answer on the back. Note that part of the point of this question is for you to understand what I am asking when I use the word "represent."
- (5) From the verbal description above, explain why doesn't $s(2)$ make sense? In other words, why is 2 not in the domain of s ?
- (6) Check your answers with another student.

- (7) Draw a number line showing the values 1, 1.9, 1.99, 2, 2.001, 2.01, 3. Draw a second number line showing the values $-1, -1.9, -1.99, -2, -2.001, -2.01, -3$ (getting them in the right order).

- (8) Fill out the following tables, using your function $s(x)$.

x approaches 2 from below. $x \rightarrow 2^-$	
x	$s(x)$
1	
1.9	
1.99	
1.999	
1.9999	

x approaches 2 from above. $x \rightarrow 2^+$	
x	$s(x)$
3	
2.1	
2.11	
2.111	
2.1111	

- (9) What value does the function $s(x)$ approach as x approaches 2 from above (as the values of x ? From below?

- (10) The number 2 is not in the domain of s , however by looking at values of x as x approaches 2, we can see what $s(x)$ approaches. In this case we do not write $s(2)$ (2 cannot be plugged into s). Instead, we write

$$\lim_{x \rightarrow 2} s(x).$$

We call this the limit of $s(x)$ as x approaches 2.

- (11) Use the following tables to determine this limit.

$$\lim_{x \rightarrow -2} \frac{x^2 + 5x + 6}{x + 2}$$

x approaches -2 from below. $x \rightarrow -2^-$	
x	$s(x)$
-3	
-2.1	
-2.11	
-2.111	
-2.1111	

x approaches 2 from above. $x \rightarrow 2^+$	
x	$s(x)$
-1	
-1.9	
-1.99	
-1.999	
-1.9999	

Put your answer here:

$$\lim_{x \rightarrow -2} \frac{x^2 + 5x + 6}{x + 2} = \underline{\hspace{2cm}}$$

- (12) What do you get when you plug in $x = -2$ to $\frac{x^2+5x+6}{x+2}$?