Newberger, Calculus 122, Fall 2005

Worksheet: Section 4.5 with trigonometric functions

Consider the function $g(x) = \sin(2x) - 2\sin(x)$.

A. Find the domain of g.

B. Find the x- and y-intercepts of g.

Tip for finding the x-intercepts:
To find the x-intercepts, you want to solve $g(x) = 0$. Here that means you want to solve

$$\sin(2x) - 2\sin(x) = 0.$$

Use a trigonometric identity to simplify this: substitute in from the double angle formula $\sin(2x) = 2\sin(x)\cos(x)$. *Now factor out a* $\sin(x)$. *Do you know what to do next? Remember if $ab = 0$ then $a = 0$ or $b = 0$. Use this principle to solve for x.*

C. Is g an even function, an odd function, or a periodic function (or none of the above)?
An even function satisfies $f(-x) = f(x)$. Its graph is symmetric across the y-axis.
An odd function satisfies $f(-x) = -f(x)$. Its graph on the positive x-axis looks like an upsidedown version of its graph on the negative x-axis.
A periodic function satisfies $f(x+c) = f(x)$ for some positive number c. Its graph repeats itself on every interval of length c. **Upshot:** Calculate $g(-x)$ to see if you get $-g(x)$ or $g(x)$. This will determine if your graph is either odd or even (it may be neither). In this section, only the trigonometric functions will be periodic.

Tip about odd and even:
Look at the graphs of $\sin(x)$ *and* $\cos(x)$, *and note that* $\sin(x)$ *is odd and* $\cos(x)$ *is even. This means that*

$$\sin(-x) = -\sin(x) \quad \textit{and} \quad \cos(-x) = \cos(x).$$

Calculate $g(-x)$ and determine if g is odd or even.

D. Find the vertical and horizontal asymptotes of g, if any.

i. Horizontal asymptotes. Find the limits as $x \to \pm\infty$ to determine the horizontal asymptotes, if any. Use L'Hospital's rule, if it applies.

Tip about horizontal asymptotes of periodic functions:
Periodic functions repeat themselves on each period, so they do not tend closer and closer to any individual number as $x \to \pm\infty$. Thus the limits of periodic functions as $x \to \pm\infty$ does not exist.

ii. Vertical asymptotes. Remember if $x = a$ is a vertical asymptote, then a is not in the domain of the function. If g has a vertical asymptote at $x = a$, find the limits as $x \to a^+$ and as $x \to a^-$.

E. Find the intervals on which g is increasing or decreasing. Include the asymptotes and the critical numbers on your number line, when finding the end points of your intervals.

Tip for finding the critical numbers:

1. *Find $g'(x)$, and check your answer is correct before going on.*
2. *Use the trigonometric substitution from the double angle formula:* $\cos(2x) = 2\cos^2 x - 1$ *to rewrite $g'(x)$ only in terms of* $\cos x$ *alone (eliminating the* $\cos(2x)$*).*
3. *To solve the resulting equation for x you will use a technique for solving quadratic trigonometric equations. First, substitute a variable, like w, for* $\cos x$ *in your equation. Note that this means you will substitute* $w^2 = \cos^2 x$ *as well. Now use the quadratic equation or factor to solve for the possible values of w. You will get two answers: $w =$ ____________ or $w =$ ____________. Now, put $w = \cos(x)$ back in. Now you have* $\cos x =$ ____________ *or* $\cos x =$ ____________.
4. *Solve each of these for x to get the critical numbers for g.*

Tips about intervals for periodic functions:

Since your function is periodic, it repeats itself. So you only need to determine what is going on in one period to know what is happening on the whole real line. Your answer for the intervals of increase should look something like this:

"The function $g(x)$ is increasing on the intervals (a, b), $(a + 2\pi, b + 2\pi)$, $(a - 2\pi, b - 2\pi)$, $(a + 4\pi, b + 4\pi)$,$(a - 4\pi, b - 4\pi)$, etc...,"

where the a and b are the endpoints that you initially studied. You can express them using the "$2\pi k$" notation if you prefer.

F. Find the local maxima and minima of g. The local maxima and minima will not occur at asymptotes, since local maxima and minima must be in the domain of the function.

G. Find the intervals on which g is concave up or concave down. Include the asymptotes as endpoints for your intervals. Find the inflection points of g; asymptotes do not count as inflection points, since inflection points must be in the domain of the function.

Tips about solving $g''(x) = 0$:

Solving $g''(x) = 0$ is similar to the techniques you used when you solved for the x-intercepts. Check that your second derivative is correct before going on.

H. Graph g. Label the coordinates of the maxima, minima and inflection points on your graph.

Tip about graphing periodic functions:

Draw the graph on one period and then copy your picture to make it repeat to the left and the right.

Done? The problem you will study in your homework is Section 4.5#36 $f(x) = \cos^2 x - 2\sin x$. If you have time, start working on this problem during the activity. Here is a useful trig identity: $\cos^2 x = 1 - \sin^2 x$ (which is the same as $\cos^2 x + \sin^2 x = 1$). You can choose to make the substitution to the original function, and leave it like that for the whole problem, if you want to.