Section 2.3 #6 *If $S \subseteq \mathbb{R}$ contains one of its upper bounds, show that this upper bound is the supremum of S.*

Assumptions: We assume that S contains one of its upper bounds. This means that there is an element (let's call it u) in S such that u is an upper bound for S. Anytime you encounter a term that we have given a formal definition, you should use that definition or the theorems and lemmas we have proved using it. So since we know that u is an upper bound for S, we know that for every $s \in S$, we have $s \leq u$.

What we want to prove: We should prove that u is the supremum for S. Refer to the formal definition of supremum. To prove that u is a supremum for S, we must show that (1) u is an upper bound for S, and (2) if v is any upper bound for S, then $v \geq u$. Since (1) is one of our assumptions, we only need to prove (2). This is an if-then statement, so assume the "if" part and prove the "then" part.

Upshot: Assume (a) u is an upper bound for S and (b) $u \in S$, and suppose in addition that (c) v is an upper bound for S. You should prove $v \geq u$.

Section 2.3 #8 *Let $S \subseteq \mathbb{R}$ be nonempty. Show that if $u = \sup S$, then for every number $n \in \mathbb{N}$ the number $u - \frac{1}{n}$ is not an upper bound of S, but the number $u + \frac{1}{n}$ is an upper bound of S.*d

To prove a "for every $n \in \mathbb{N}$" statement, prove it for a fixed n (but do not specify a particular example), and then say that your proof will hold for all of them, since the only assumption you used about n was that it was in $\mathbb{N}$.

Since upper bound and supremum are words that we have formally defined, you should use the definitions, or the theorems and lemmas that we proved using the definitions. In particular, to prove that $u + \frac{1}{n}$ is an upper bound, you must show that it satisfies a definition of upper bound. In other words, you must show that for every $s \in S$, we have $s \leq u + \frac{1}{n}$.

Upshot: Assume $u = \sup S$. Let $n \in \mathbb{N}$. Use the definitions of supremum and upper bound to show that $u - \frac{1}{n}$ is not an upper bound and $u + \frac{1}{n}$ is an upper bound.