Newberger Math 361A Spring 03

Section2.2 #4 and #5

There are many ways to do these problems. I am going to guide you to complete them in a particular way, but you are welcome to revise your own methods, if you prefer. The most common mistake students made on these problems was applying Theorem 2.2.2(c) and making the wrong conclusion. These problems deal with strict inequalities, however the theorem only gives you only “less than or equal,” so though you can apply the theorem, it does not give you the conclusion you want.

One student had the good idea to prove a lemma and then to use it in both problems #4 and #5. My suggestions below are based on that idea. Again, this is my suggested method, but not the only way; you may, instead, prove the statements in the problems “directly,” i.e. without using theorems and lemmas.

So, I suggest you begin your homework with the following Lemma:

Lemma. *Let $c \geq 0$, $d \in \mathbb{R}$. Then $|d| < c$ if and only if $-c < d < c$.*

Your proof can be very similar to the proof of 2.2.2(c), and I suggest that you use 2.2.2(c) to guide your argument. Here is a brief outline. Fill in the details on your homework.

Proof. “$\Rightarrow$” Assume that $|d| < c$. Prove that $-c < d < c$. Prove it in three cases.
Case 1. Suppose $d < 0$. Then $|d| = -d$ so our assumptions become $-d < c$. We want to prove that $-c < d < c$.
Case 2. Suppose $d = 0$. Then $|d| = 0$ so our assumptions become $0 < c$, and we want to prove $-c < 0 < c$.
Case 3. Suppose $d > 0$. Then $|d| = d$ so our assumptions become $d < c$. We want to prove $-c < d < c$.

“$\Leftarrow$” Assume that $-c < d < c$. Prove that $|d| < c$. This is probably easiest in three cases as above. Here we assume $-c < d < c$ in all three cases.
Case 1. Suppose $d < 0$. Then $|d| = -d$, so we want to prove $-d < c$.
Case 2. Suppose $d = 0$. Then $|d| = 0$, so we want to prove $0 < c$.
Case 3. Suppose $d > 0$. Then $|d| = d$, so we want to prove $d < c$.

□

Now I will leave it to you to figure out what to put for c and d to use the Lemma to prove your homework problems. Both #4 and #5 can be proved using this lemma.