Newberger Analysis 361A Fall 03

Homework Worksheet

Let $(x_n : n \in \mathbb{N})$ and $(y_n : n \in \mathbb{N})$ be sequences of real numbers.

(1) Suppose $\lim x_n = 3$. Prove that there exists a number K such that if $n \geq K$, then $\frac{3}{2} < |x_n|$.

(2) Generalize the previous problem, by replacing 3 by any nonzero number. i.e. Suppose $\lim x_n = x$ and $x \neq 0$. Prove that there exists a number K such that if $n \geq K$, then $\frac{|x|}{2} < |x_n|$.

(3) Suppose $\lim x_n = x$ and $x \neq 0$. Use (2) to prove that there exist numbers K and M, such that if $n \geq K$, then $\frac{1}{|x_n|} < M$.

(4) **(Due Monday 10/13)** Prove that if $\lim x_n = x$ with $x \neq 0$, and $(x_n y_n : n \in \mathbb{N})$ is a bounded sequence, then $(y_n : n \in \mathbb{N})$ is a bounded sequence.

(5) Suppose that $x, q \in \mathbb{R}$ with $x \neq 0$. Prove that

$$\left| y_n - \frac{q}{x} \right| \leq \frac{|y_n|}{|x|}|x - x_n| + \frac{1}{|x|}|x_n y_n - q|.$$

(6) **(Due Wednesday 10/15)** Suppose that $\lim x_n = x$ with $x \neq 0$, and $\lim x_n y_n = q$. Prove that $\lim y_n = \frac{q}{x}$.

(7) **(Due Friday 10/17)** Show that the condition $x \neq 0$ in (6) is necessary. In other words give an example of a sequence $(x_n : n \in \mathbb{N})$ with $\lim x_n = 0$ and a sequence $(y_n : n \in \mathbb{N})$ such that $(y_n : n \in \mathbb{N})$ does not converge, but $(x_n y_n : n \in \mathbb{N})$ does converge.