

### Exam 3 Review

This exam covers material from the subsection of Section 2.4 called Density of rational numbers in  $\mathbb{R}$  (page 42-43), Sections 3.5, 3.7, 4.1 and 5.1.

- (1) You should be familiar with all of the following vocabulary. You **will be asked** to give precise mathematical statements for some of the vocabulary that appears below.
  - **Cauchy sequence**, Definition 3.5.1 (page 81).
  - **Series convergence**, Definition 3.7.1 (page 89).
  - **Cluster point**, Definition 4.1.1 (page 97).
  - **Limit of a function**, Definition 4.1.4 (page 98).
  - **Continuous at a point**, Definition 5.1.1 (page 120).
  - **Continuous on a set**, Definition 5.1.5 (page 121).
- (2) You should be familiar with all of the following theorems. The proofs provide examples on which you can base your own proofs, and you may call on these theorems in your arguments.
  - The Density Theorem 2.4.8 and Corollary 2.4.9 (page 42).
  - The Cauchy convergence criterion, Theorem 3.5.5 (See also 3.5.3 and 3.5.4, page 82).
  - Geometric series, Example 3.7.2(a) (page 90).
  - $n^{\text{th}}$  term test 3.7.3, (page 91).
  - p-series, Example 3.7.6(d) (page 92).
  - alternating harmonic series, Example 3.7.6(f) (page 92).
  - Comparison test 3.7.7 (page 93).
  - Limit comparison test
  - Theorem 4.1.2 (page 97).
  - Uniqueness of limits, Theorem 4.1.5 (page 99).
  - Sequential criterion, Theorem 4.1.8 (page 101).
  - Divergence criterion, Theorem 4.1.9 (page 102).
  - Sequential criterion for Continuity, Theorem 5.1.3 (page 121).
  - Discontinuity criterion 5.1.4 (page 121).
- (3) (a) *Take home part of the exam:* Prove the sequential criterion, Theorem 4.1.8 (page 101). Make sure that your proof is complete and mathematically correct. Due Monday 12/1.  
(b) On the in class exam you will be asked to prove the Comparison Test, Theorem 3.7.7.
- (4) About the exam:
  - 40% of the exam involves proving theorems listed in (3) and stating definitions listed in bold in (1) above.
  - You will be asked to prove at least one statement under the assumption that a limit exists or that a function is continuous.
  - You may be asked to give examples of functions that are continuous or discontinuous at given points and/or functions whose limits exist or do not exist at given points (for example at the rational numbers).
  - You may be asked to give counterexamples for false statements about series. You may be asked to prove statements about series that follow from the theorems.
  - You may be asked to prove a given limit does or does not exist, and/or a given sequence is or is not cauchy.