

Exam 2 Review

This exam covers material from Sections 3.1, 3.2, 3.3 and 3.4.

- (1) You should be familiar with all of the following vocabulary. You **will be asked** to give precise mathematical statements for some of the vocabulary that appears in boldface type below.
 - **limit of a sequence**: Definition 3.1.3.
 - **bounded sequence**: Definition 3.2.1.
 - monotone, **increasing**, and decreasing: Definition 3.3.1.
 - subsequence: Definition 3.4.1.
- (2) You should be familiar with all of the following theorems. The proofs provide examples on which you can base your own proofs.
 - Theorem 3.1.4: Uniqueness of limits.
 - Theorem 3.2.2: A convergent sequence of real numbers is bounded.
 - Theorem 3.2.3(a): Suppose that (x_n) and (y_n) are sequences of real numbers that converge to x and y respectively. Then
 - (i) $(x_n + y_n)$ converges to $x + y$,
 - (ii) $(x_n - y_n)$ converges to $x - y$,
 - (iii) $(x_n y_n)$ converges to xy , and
 - (iv) (cx_n) converges to cx .
 - Theorem 3.2.7: Squeeze Theorem.
 - Theorem 3.3.2: Monotone Convergence Theorem.
 - Theorem 3.4.7: Monotone sequence theorem.
 - Theorem 3.4.8: Bolzano-Weierstrass Theorem.
- (3) You will be asked to give a proof of
 - One part of Theorem 3.2.3(a), and
 - Theorem 3.3.2: Monotone Convergence Theorem (for increasing sequences).
- (4) About the exam:
 - (a) 40% of the exam involves proving theorems listed in (3) and stating definitions listed in bold in (1) above. You are not required to use the exact words given in the book or in lecture, but your definitions and statements must be correct, precise mathematical statements. I suggest memorizing them.
 - (b) You will be asked to show that you know how to prove statements involving limits. For example:
 - (i) You will be asked to prove that a sequence converges to a particular number, like in problems #5 and 6 on page 60.
 - (ii) You will be asked to prove a statement under the assumption that particular limits converge. See for example Theorem 3.2.3, Problems #3,4 on page 67, Theorem 3.1.4 and Theorem 3.2.7.
 - (c) You will be asked short answer questions about sequences and subsequences, to show you know how to make relationships between the various vocabulary words. Pay particular attention to convergence, boundedness, monotone sequences and subsequences. This may involve giving examples of sequences with certain properties, or answering true or false questions.