Newberger Math 444 Fall 2001

MIDTERM EXAM REVIEW

The exam will cover the following parts of Chapter 2 and 3: Section 2.1, 2.3, 3.1 and 3.2, as well as Example 3.6.1, Example 3.6.2, Definition 3.6.3, Definition 3.6.5, Proposition 3.8.1, Definition 3.8.2, Example 3.8.1 and Example 3.8.3.

(1) The following list consists of terms that you may be asked to define and theorems that you may be asked to state (and know by name).
- Composition of functions: Definition 2.1.2 p49
- One-to-one, onto and bijection (the book calls a bijection a "one to one correspondence"): Definition 2.1.4 p51
- Identity function: Definition 2.1.7 p53
- Permutation and S_n: Definition 2.3.1 p64
- Cycle of length k: Definition 2.3.2 p66
- Disjoint cycles: Definition 2.3.3 p68
- Transposition: Definition 2.3.9 p73
- Even and odd permutations: Definition 2.3.12 p75
- Binary operation: Definition 3.1.1 p80
- Group: Definition 3.1.3 p82
- Abelian group: Definition 3.1.8 p86
- Order of a group, $|G|$: Definition 3.1.9 p87
- Subgroup: Definition 3.2.1 p92
- Subgroup generated by an element, $< a >$, and cyclic group: Def 3.2.5 p95
- Order of an element, $o(a)$: Definition 3.2.7 p97 (Also, order of a permutation: Definition 2.3.6 p71)
- Lagrange's Theorem: Theorem 3.2.10 p99
- Dihedral group, D_n: Definition 3.6.3 p131
- Alternating group, A_n: Definition 3.6.5 p133
- Left and right coset: Definition 3.8.2 p147

(2) The following list consists of Lemmas, Propositions, Theorems and Corollaries with which you should be familiar. The asterisk (*) indicates that you may be asked to give the proof on the exam.

Proposition 2.1.3(p50), Proposition 2.3.8(p72), Proposition 2.3.10(p73), Theorem 2.3.11(p74), Proposition 3.1.2(p81), Proposition 3.1.5(p84; Note that "Sym(S)" is the set of bijections from S to itself), Proposition 3.1.6(p85), Proposition 3.1.7(p85), Proposition 3.2.2p(93), Corollary 3.2.3(p94), Corollary 3.2.4*(p94), Proposition 3.2.6(p96), Proposition 3.2.8(p98)*, Lemma 2.3.8(p98), Theorem 3.2.10(p99), Corolary 3.2.11(p100), Corollary 3.2.12*(p100), Proposition 3.8.1(p146)*

(3) Be familiar with the following examples of groups.
$(\mathbb{Z}_n, +)$, $(\mathbb{Z}, +)$, $(\mathbb{Q}, +)$, $(\mathbb{R}, +)$, $(\mathbb{Z}_n^{\times}, \cdot)$, $(\mathbb{Q}\backslash\{0\}, \cdot)$, $(\mathbb{R}\backslash\{0\}, \cdot)$, S_n, A_n, D_n, $Gl(n, \mathbb{R})$.

(4) **Guarantees.** Your exam will have from 4 to 6 problems.
- (40%) You will be asked to state some of the definitions and theorems listed in (1) above and to prove one or two of those marked with an asterisk in (2).
- (10%) You will be given a new definition and asked to prove something, state examples, or answer questions about it.
- (50%) Prove statements, answer questions and provide examples from the material above, including the following.
  - You will be given a set and a binary operation and asked to prove it is a group.
  - You will be given a group and a subset and be asked to prove it is a subgroup. (For these, use Proposition 3.2.2, or Corollaries 3.2.3 or 3.2.4.)
  - In order to solve some of the problems, you will be required to do computations in S_n, similar to those on the quiz.
  - You will be asked to compute a subgroup generated by a given element of a given group (i.e. Compute $< a >$).
  - You will be asked to compute cosets of a given subgroup in a given group.