

CUMULATIVE FINAL EXAM REVIEW

The cumulative part of the exam will cover all the material on the other 3 review sheets. This part of the exam will have 4 problems.

Vocabulary The following list consists of terms that you may be asked to define and theorems that you may be asked to state (and know by name).

- Divisor, multiple, divides: Definition 1.1.1 p5
- $n\mathbb{Z}$: p5
- Greatest common divisor: Definition 1.1.5 p8
- Relatively prime: Definition 1.2.1 p16
- Prime: Definition 1.2.4 p17
- Least common multiple: Definition 1.2.8 p20
- Congruence modulo n : Definition 1.3.1 p24
- Congruence class of a modulo n : Definition 1.4.1 p32
- \mathbb{Z}_n : p33
- Permutation: Definition 2.3.1 p64
- Even and odd permutations: Definition 2.3.12 p75
- Group: Definition 3.1.3 p82
- Abelian group: Definition 3.1.8 p86
- Order of a group, $|G|$: Definition 3.1.9 p87
- Subgroup: Definition 3.2.1 p92
- Subgroup generated by an element $\langle g \rangle$, and cyclic group: Def 3.2.5 p95
- Order of an element, $o(g)$: Definition 3.2.7 p97
- Lagrange's Theorem: Theorem 3.2.10 p99
- Left and right coset: Definition 3.8.2 p147
- Direct product: Definition 3.3.3 p105.
- Group homomorphism: Definition 3.7.1 p137.
- Group isomorphism: Definition 3.4.1 p112.
- Kernel of a group homomorphism: Definition 3.7.3 p139.
- Ring with identity: Definition 5.1.1 p197.
- Commutative ring with identity: Definition 5.1.1 p 197.
- Subring: Definition 5.1.2 p200.
- Unit (and the notation R^\times): Definition 5.1.4 p 202
- Zero divisor: p202
- Integral domain: Definition 5.1.6 p203.
- Field: Definition 3.3.5 p107.
- Ring homomorphism and isomorphism: Definition 5.2.1 p208.
- Kernel of a ring homomorphism: Definition 5.2.4 p213.

Guarantees.

- You will have questions involving \mathbb{Z}_n . The worksheet is a source of good examples of these. Be sure you are familiar with \mathbb{Z}_n , \mathbb{Z}_n^\times and $\mathbb{Z}_m \times \mathbb{Z}_n$. You may be asked about subgroups, homomorphisms (and their kernels) and isomorphisms of these groups, or to calculate order, or to find units, inverses and zero divisors, or to list cosets of particular subgroups.

- You will have questions involving S_n . The quiz and second midterm are a source of good examples of these, also the Solved Problems on the web from Section 2.3. You may be asked about subgroups, homomorphisms (and their kernels) and isomorphisms of these groups, or to calculate order, inverses or to list cosets of particular subgroups. You may also be asked about the subgroup A_n of even permutations.

- You will either be asked to prove a given set is a group or to prove a given set is a ring. It is likely that you will be asked to prove a given subset of a group (or a ring) is a subgroup (or a subring).

- You will be asked about homomorphisms and isomorphisms (of both groups and rings). For example you may be asked to find homomorphisms or isomorphism, or to check if a given function is a homomorphism or isomorphism. You are likely to have questions the kernel and image of homomorphisms.