

Law of Large Numbers and The Monty Hall Problem by Rebecca Long

There are two goals for this lesson. The first is to have students learn the Law of Large Numbers and the second is to have the students use the Law of Large Numbers to determine the correct answer to the Monty Hall problem. You can do activities with the students to accomplish these two goals. This lesson can be used with a range of grade levels, from 7th all the way to 11th. The overall concepts are very easy and could be understood by 7th graders. On the other hand, this activity could be used in a probability class in high school, in which case the students would go into a lot more depth about the mathematical reasons and proofs of the theorems and the answer to the Monty Hall problem. This lesson may take 1-3 days depending on how much depth you go into it with your students. You can vary the activities and questions to meet the level of your students.

The first activity is to help the students discover the Law of Large Numbers. To do this you can do a coin flipping activity. The students work in pairs and each flip a coin 10 times. They should record each time the coin lands on heads and each time the coin lands on tails. The students will expect to get heads half of the time and tails half of the time, however, this may not be the case. You should collect and analyze the students' and class' data. You can then ask the students some leading questions in order to help them determine the Law of Large Numbers. Some possible leading questions are: why are their individual percentages of heads not necessarily 50%? Why is the class data closer to the correct result? What would the students have to do to get a more accurate result? And finally, what do the students think the Law of Large Numbers states? Hopefully the students would be able to answer the last question by the time you got to it. You could then give the students the actual theorem, which is: the percentage of the time an event occurs in n trials of an experiment tends to the probability of the event as n grows.

After the students have learned and understand this concept, you can move onto the Monty Hall problem. First, introduce the problem to the students. The problem is the following: you are given three doors to choose from, two doors contain goats behind them and one door contains a prize behind it. You choose one of the doors and then the host, who knows what is behind each door, shows you one of the other incorrect doors. You then are given the option to stay with your door, or switch to the other door. The question is whether it is to your advantage to switch or should you stay with your original door. This is counter-intuitive, but you should always switch.

You should let the students think about the possible answers and decide which one they think is correct, based only on logical thinking. You can then do a simulation with plastic eggs and a penny and have the students collect data. One student will place a penny under one of three plastic eggs. The other student will guess which one they think is correct. The host will show an incorrect egg, but not the one that was chosen. The other student will then be allowed to switch eggs or stay with their original choice. The students will record each time they either stay or switch and will record when they win. They will not record when they lose. The students can play this several times. Then collect and analyze the individual and class data. Ask the students what they should do in order to make their results more accurate.

Then, if possible, have the students use simulation websites on the Internet. Some good ones are: <http://math.ucsd.edu/~crypto/Monty/monty.html>, <http://147.4.150.5/~matsrc/MontyHall/MontyHallSim.html>, and <http://www.stat.sc.edu/~west/javahtml/LetsMakeaDeal.html>.

These websites have very large sample sizes, and due to the Law of Large Numbers, their results are very close to the actual probability. During this activity, you can keep track of what the students think the correct answer is. You can take a vote at the beginning of the class and then allow the students to change their mind at any point after each activity.

In conclusion, this lesson would be very valuable for students at many different levels. The Monty Hall problem is counter-intuitive and difficult to understand, however, with the activity and the convincing Law of Large Numbers theorem, it is easier to remember and grasp.