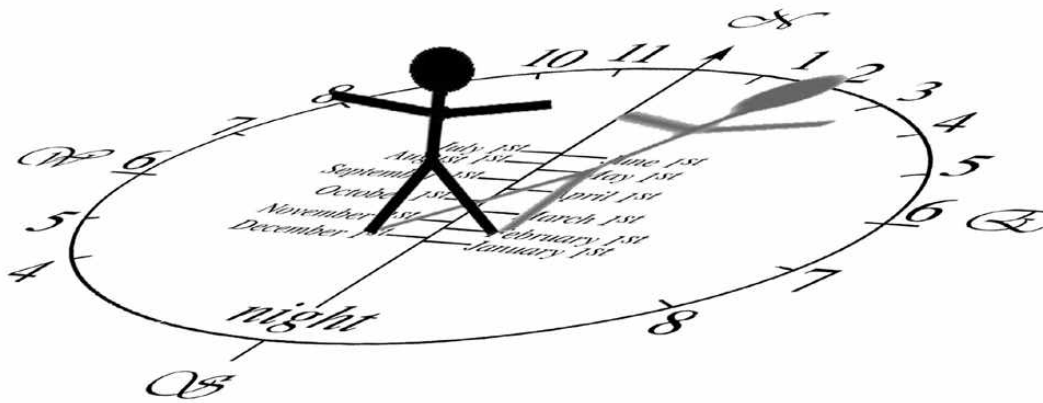


Making an Analemmatic Sundial by Patrick Kehan

Up until the invention of the mechanical clock, mankind has used many different sundials to read time. In this activity students will construct an analemmatic sundial, which is a horizontal sundial with an elliptical shape in which the shadow-casting object, or gnomon, is moved according to the date. In this project the sundial will be constructed on the pavement with chalk or paint, and a person, standing according to the date on the face of the dial, will cast the shadow. We begin by finding the latitude and deciding the height of the gnomon. Using this information, we construct an ellipse of the appropriate size and label the hour marks and the places to stand. If desired by the instructor the mathematics needed in this project can be varied to according to the level of the students. For example, using trigonometry the students can derive the formulas for the x and y values which gives the location of each hour mark in terms of the hours angle; and the formula giving the distance to stand from the origin on a specific day in terms of latitude and declination of the sun. In all cases the students must be able to evaluate trigonometric functions and understand the construction of an ellipse.



Steps to Construct a Sundial

1. Declination of the sun

Our first step is to find the angle at which the sun's rays hit the plane of the equator at various days throughout the year. This angle, called the declination of the sun and denoted θ , is necessary to determine the point at which to stand on a given day to read the time on the sundial. See Figure 1.

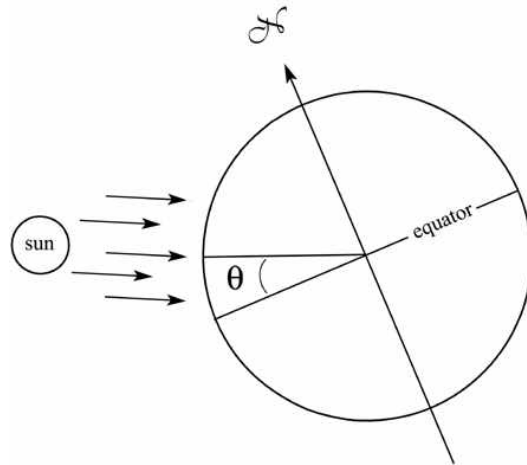


Figure 1: The declination of the sun.

Since the Earth rotates around the sun in an elliptical orbit these angles are constantly changing. For our sundial, we choose to indicate where to stand only on the first of each month; more points would make the diagram too cluttered and fewer points would make the time too inaccurate. The table below shows the declination of the sun on the first of each month. We will use these in Step 5.

Table 1: Declinations of the sun.

Date	Angle(θ)	Date	Angle(θ)
January 1	-23.13	July 1	23.00
February 1	-17.30	August 1	18.00
March 1	-8.00	September 1	8.50
April 1	4.25	October 1	-2.90
May 1	15.00	November 1	-14.00
June 1	22.00	December 1	-23.40

Step 2. Finding your latitude

The next step in creating the analemmatic sundial is finding the latitude where our sundial will be used, which we will denote by ϕ (see Figure 2). There are a variety of tools that can be used to measure latitude, such as the quadrant, an ancient tool for navigation, the Global Positioning System (GPS), a more modern tool for navigation with the assistance of satellites, or simply a map. However, the easiest and fastest way to pinpoint your latitude is probably with the assistance of the internet. One site for instance, <http://www.zipinfo.com/search/zipcode.htm> will give your latitude when you enter your zip code.

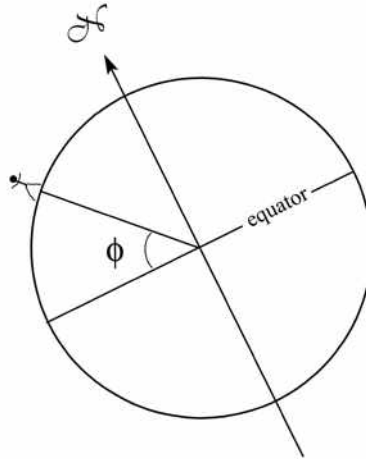


Figure 2: Latitude.

Step 3. Constructing an ellipse

Now the third step is creating the ellipse. It must be outside in a location that receives full sun. First we will decide the appropriate size for the ellipse, and then draw it on the pavement with paint, chalk or heavy-duty tape. The dimensions of the ellipse depend on the height of the shadow-casting object (see Figure). Since a person will cast the shadow, and all people are not the same height, we must decide what height will accommodate the most people. Ideally, the ellipse is sized so that the shadow reaches the hour marks on the perimeter.

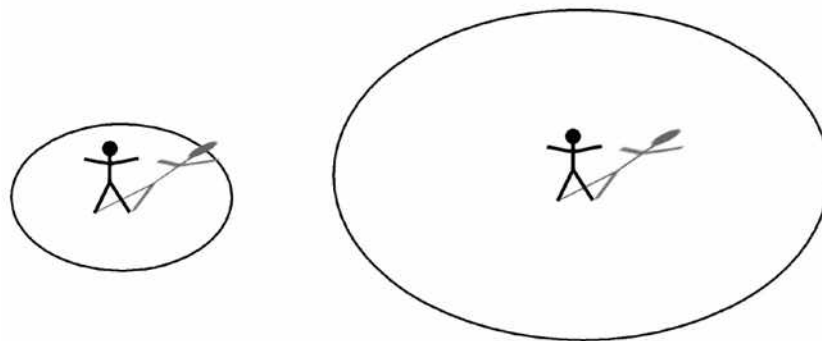


Figure 3: On the left, the ellipse is the right size for Stickman's height. On the right, it is sized for someone taller.

To accommodate a class of students, we suggest using the smallest person's height to size the ellipse. If necessary, shorter people can lengthen their shadow by holding their arms overhead or by standing up on a stool or a chair.

Once we chose a height, we find the lengths of the major and minor axes and the focal points of the ellipse (see Figure 4). The following formulas give half the length of the major (East-West) and minor (North-South) axes denoted a and b respectively.

$$b = (\text{height}) \tan(\phi)$$

$$a = (\text{height}) \sec(\phi) = b / \sin(\phi)$$

where ϕ is the latitude. The focal points, F_1 and F_2 , will be located on the East-West axis at a distance $(a^2 - b^2)^{1/2}$ on either side of the center of the ellipse.

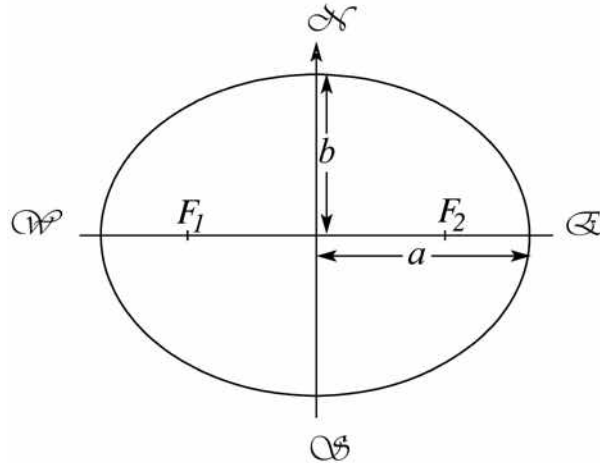


Figure 4: The ellipse.

Now we can construct the ellipse. Choose a center for the ellipse and draw East-West and North-South axes, marking the focal points on the East-West axis. Take a string of length $2a$ and attach each end to a focal point. With a piece of chalk pull the string taut and move it along, continuing to keep the string taut until an ellipse is completed.(see Figure 5).

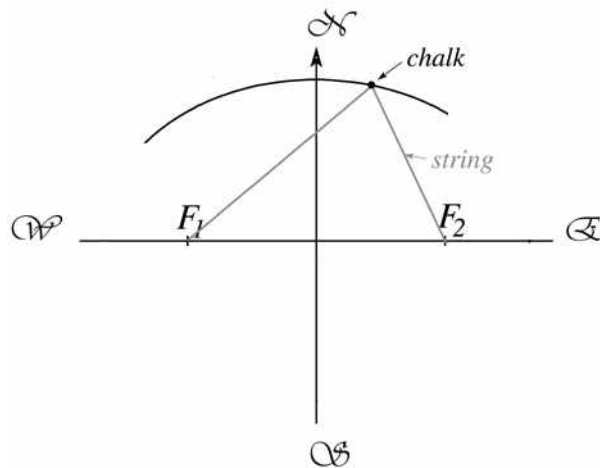


Figure 5: Constructing an ellipse.

Step 4. Labeling hour points

This step consists of labeling hour points around the edge of the ellipse. We mark the hours every 15 degrees around the edge of the ellipse. The x and y coordinates for each hour mark can be found from the formulas:

$$x = a \sin(\lambda)$$

$$y = a \sin(\phi) \cos(\lambda) = b \cos(\lambda),$$

where $\lambda = 0, 15, 30, 45, \dots, 345$. See Figure 6.

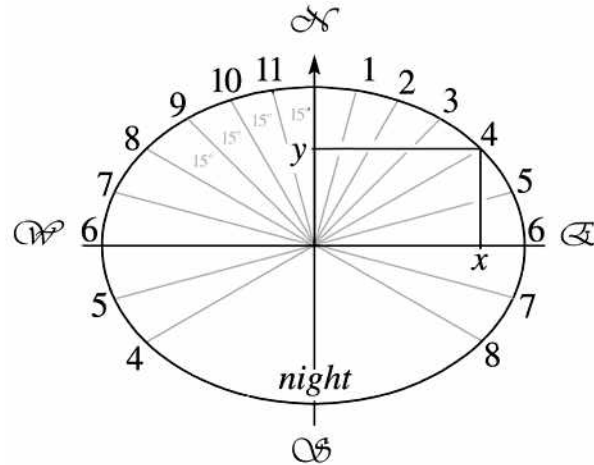


Figure 6: The hour marks are every 15 degrees.

Step 5. Making the scale of dates.

In this step we mark the places to stand during various times of the year on the face of the sundial in order to read the time from our shadow. This information depends on the declination of the sun, which can be found from the table in Step 1. Once again, when labeling the scale of dates we make a mark for the first of every month, since marks for all 365 days would be too confusing to read. From the origin of the ellipse measure a length of S along the North-South axis, where

$$S = a \tan(\theta) \cos(\phi),$$

and where θ is the declination of the sun given in Step 1 (See Figure 7).

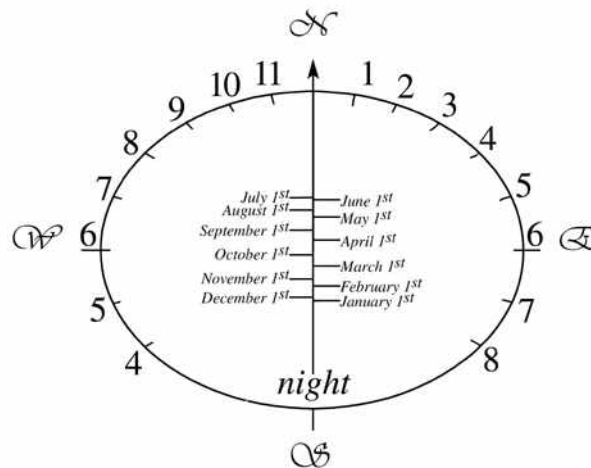


Figure 7: The completed sundial.

Once all the steps have been completed, your analemmatic sundial is ready to tell time. Hope you have some sunny days!

References:

Analemmatic Sundials: How to build on and why they work by Chris Sangwin and Chris Budd.
<http://pass.maths.org.uk/issue11/features/sundials.html>.

Make an Analemmatic garden Sundial <http://www.cyberspace.org/~jh/dial/analemmatic.html>