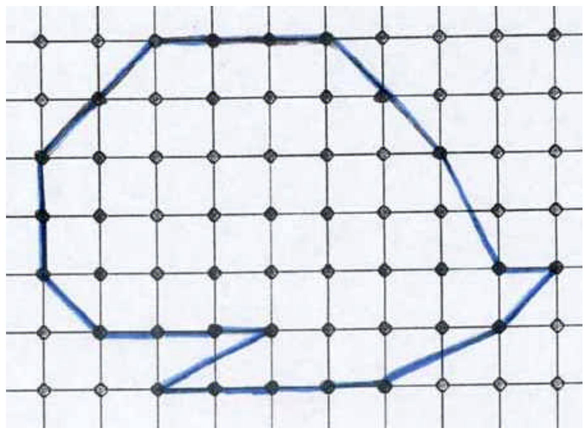
Pick's Theorem

by Marisa E. Ialongo

The topic that I chose to present at the Pennsylvania Council of Teachers of Mathematics was Pick's Theorem. I presented Pick's Theorem to Mathematics teachers as a discovery lesson that they could use to lead their students in finding the area of a regular polygon. The duration of my presentation was 10 -15 minutes.

I began the presentation with a transparency of grid points with a regular polygon drawn on them. This polygon had all of its vertex points lying on the points of the grid. I presented this overhead in the beginning of my presentation so that I could show the audience an example of where Pick's Theorem can be applied. I did not give the answer at this point, but explained that we would come back to it later.



Next, I discussed how we might go about teaching the students to find the area of polygons. Most often we would have the students add up the number of blocks that are shaded in and come up with an estimate of what the area might be through this method. At this time, I stated that with our knowledge of Pick's Theorem, we area able to find the exact area of a polygon in a matter of seconds.

In my presentation, I started out with a series of overheads, which included a chart and a grid with a shaded polygon. The polygons were small and simple shapes at first, then throughout the series of overheads they became larger and more complex shapes. The chart on each overhead contained four columns; B (Boundary points), B/2, I (Interior points), and A (Area). Prior to giving my presentation I filled out all of these columns with the correct numbers for that polygon which was being displayed at the time. The idea here is to help students find patterns and recognize a formula satisfying all of the numbers in the chart.

B	B/2	I	A
4	2	0	1
8	4	1	4
14	7	4	10
18	9	10	18
10	5	0	4
12	6	4	9
18	9	12	20
22	11	7	17

By this time, I was able to present the formula for Pick's Theorem, which is

A = B/2 + I – 1.

Most everyone at my session was able to figure this out beforehand. Now it was time for me to present an application of Pick's Theorem.

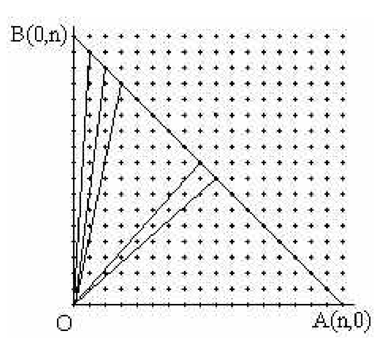
The example that I used was one that I found off of the web. The problem was rather difficult; however, it was definitely solvable. I went over this problem as part of my presentation and all the attendees seemed to understand.

Problem 1

The line connecting two points A(n,0) and B(0,n) has a simple equation, x + y = n. Therefore, it contains all the points in the form (i, n-i), i an integer. There are n-1 such points between A and B.

Connect each one of them with the origin O. The lines divide OAB into n small triangles. Superimpose an integer grid on the diagram. The question is about the number of grid points that lie inside small triangles. Obviously, the two triangles next to the axes contain no grid points in their interiors.

Prove that, for n prime, each of the remaining triangles contains exactly the same number of grid points.



Answer to Problem 1:

First, we can show that the lines hit no grid points. This has to do with n prime. Next, since the shape is triangular, we are able to find the base of the triangle, the square root of two. This is the same for all of the small triangles. These small triangles will also have the same height, which is n divided by the square root of two. Therefore, our area for all of the triangles is the same, n divided by two.

So for all triangles, the Area (according to Pick's Theorem) is n/2, which equals the Interior points + 3/2 – 1. The value of the interior points remains constant and we have proved the area is the same for all of the small triangles.

To close, I posted the original overhead that I used in the very beginning of the presentation. Next, I asked the audience to find the area of this regular polygon by using Pick's Theorem. In a matter seconds, we found that the area of the polygon was 37. I thanked everyone for coming and that completed my presentation.