What's so Magical about Magic Squares?

by Kim Denlinger

My presentation consisted of a project for students to take a look at the arrangement of numbers in magic squares to discover various number patterns and relationships. The examples are a way for students to have fun and enjoy math while still applying knowledge. The project can be introduced by: giving a description of magic squares, presenting detailed examples of a 3x3 magic square, demonstrating the construction of an odd ordered magic square, and then correlating the findings to a calendar.

A *magic square* consists of a number of integers arranged in the form of a square, so that the sum of each row, column, and main diagonal is equal. A magic square of order n is an arrangement of the numbers $\{1, 2, \ldots\ldots n^2\}$ in an nxn square. The sum of one row, column, or diagonal is $nS = 1 + 2 + \ldots. + n^2$ or $S = n(n^2 + 1)/2$ which is called the sum equation. Note that a Magic square will remain "magic" if a number is added/subtracted or multiplied/divided by every cell of the square.

There are many ways to apply magic squares in the classroom. One method is to give students a grid (particularly a 3x3 magic square) and have them try to figure out the placement of the numbers. This could be done before or after the definition is given to the students. Depending on the level of the students, they could try to figure out the different qualities of magic squares. To figure out the magic square, students could guess and check but it would best if they could see number patterns and relations. The students could then put their answers on the board and compare and contrast the different squares. The students could be asked questions that will guide their thinking to see the different number relationships.

There are four distinct patterns seen in a 3x3 magic square. The first is that the sum of each row, column, and diagonal will be 15. This can be figured out through the sum equation. The next is that the center square will always be 5. The center square is connected to one row, one column, and two diagonals. It also connects to every other square once. Algebra can be applied to these facts to determine that the center number will always be 5. Complimentary pairs are another pattern. The two squares opposite the center square will always equal 10 and each square (besides the center square) will be part of a complimentary pair. The last pattern is that the even numbers will always be placed in the corners. Write out the different combinations of three numbers that can be added together to equal 15. Each even number occurs in three different equations. The corner squares are always connected to three different sums (one row, one column, and one diagonal) so therefore the even numbers must be placed in the corners. These different number relations and patterns are shown below:

a	b	c
d	e	f
g	h	i

- Sum equals 15

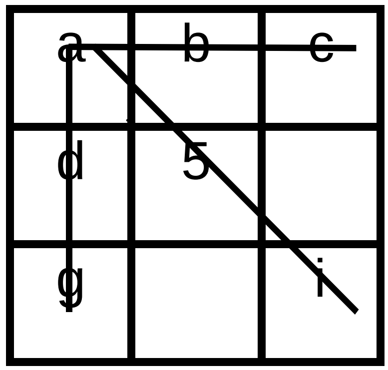
- $(a+e+i)+(b+e+h)+(c+e+g)+(f+e+d) = 60$

  $3e + (a+b+c+d+e+f+g+h+i) = 60$

  $3e + 45 = 60$ thus, $e = 5$

- $a+i = b+h = c+g = d+f = 10$
  (complementary pairs)

- even numbers (2,4,6,8)
  must be placed in the corners



1 + 5 + 9

1 + ***6*** + ***8***

3 + ***4*** + ***8***

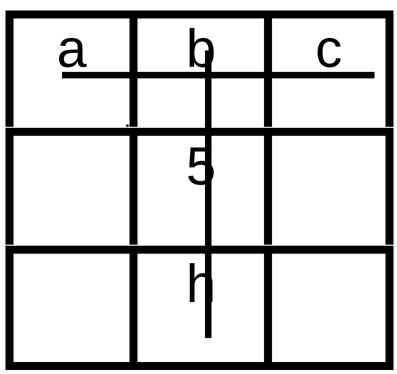
3 + 5 + 7

2 + ***4*** + 9

2 + 5 + ***8***

2 + ***6*** + 7

4 + 5 + ***6***



Once the class discusses and discovers the different number patterns and relations, they can see that the other arrangements of a 3x3 magic square can be developed by rotations and reflections. There is a total of 8 different possible arrangements of a 3x3 magic square. You could challenge the students to find the 8 different squares, and then have the students relate the number relations and patterns to the different squares. The 8 possible 3x3 magic squares are:

8	1	6
3	5	7
4	9	2

6	1	8
7	5	3
2	9	4

8	3	4
1	5	9
6	7	2

2	7	6
9	5	1
4	3	8

4	3	8
9	5	1
2	7	6

6	7	2
1	5	9
8	3	4

2	9	4
7	5	3
6	1	8

4	9	2
3	5	7
8	1	6

There is a method to construct and odd ordered magic square. This can be taught to the students after they analyzed the properties of the 3x3 magic squares. De la Loubere found the method in the 17th century. It is probably to difficult for the students to discover, but the pattern could be taught to the students, and then they could try the pattern on any odd ordered magic square. The steps to the method and a visual display is shown below:

- Place a 1 in the Middle Square of the Top Row

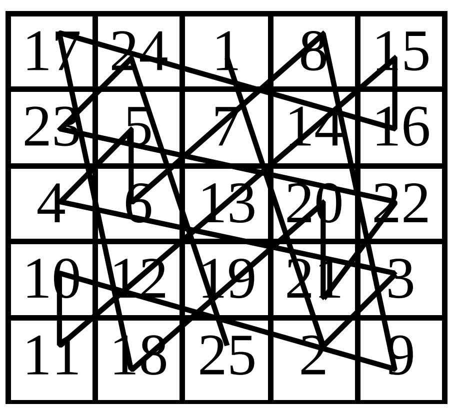
- Successive integers are placed in order along a diagonal line which slopes upward and to the right, with the following conditions:

  - *When the top row is reached, the next integer is put in the bottom row and one column to the right.*

  - *When the right-hand column is reached, the next integer is put in the left-hand column and up one row.*

  - *When a square is reached which already has a number in it, the next integer is placed in the square immediately below the last square, which was filled.*

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9



Now that the students have explored many areas of magic squares, they can investigate other fun and interesting ways to apply what was discovered. One key example is in some patterns with calendars. Each student could receive a calendar of a different month or the class could be split into 12 groups so each group has a different month. Allow the students time to explore and discover patterns in the numbers in a calendar and the relationship to magic squares. Below is an example of a way to make a correlation between magic squares and a calendar.

S	M	**T**	**W**	**Th**	F	**S**
				1	2	3
4	5	6	***7***	***8***	***9***	10
11	12	13	***14***	***15***	***16***	17
18	19	20	***21***	***22***	***23***	24
25	26	27	28	29	30	

- *Choose a date, n, to represent the middle number.*
- *Figure out the sum of the diagonals, rows, and columns that surround n.*
- *Apply algebra: (example from above)*

$$n + (n+a) + (n-a) = S$$

$$3n = S$$

The students can continue to discover different patterns with calendars and magic squares. Hopefully this application of magic squares will be a simple form of recreational math for the students. Yet, the magic squares can be looked at more in depth to develop more mathematical knowledge. It is a great activity for students to enjoy the fun of mathematics while continually discovering new ideas with numbers.