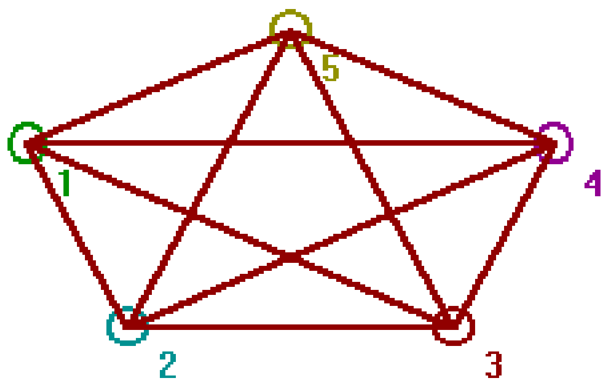
Scheduling tournaments
by Bill Sechrist

This idea for a lesson focuses on introducing students to the mathematics involved in scheduling a round robin tournament. It provides students: a real world problem that applies theory, the ability to use computers to develop conceptual understanding, and the ability to work in groups and discuss methods of approaching and solving problems. The lesson can be done either in the classroom or in the computer lab.

The teacher should begin by explaining the mathematical ideas surrounding scheduling a round robin tournament and gauge student's understanding throughout the discussion. A **round robin tournament** is a tournament with the property that every team plays every other team exactly once during the tournament. The graph for a round robin tournament with n teams is the complete graph on n vertices, or K_n.

For example: Here is a complete graph of K_5



The total number of games played is the number of edges on the graph K_n. The total number of games turns out to be $\binom{n}{2}$ or $\left(\frac{n!}{2!(n-2!)}\right)$.

In tournament designs the number of teams participating has an effect on the schedule making process. First we will cover the case that the number of teams participating is odd. If the number of teams, n, participating in the tournament is odd then it is possible to complete the tournament in n rounds. Each team will play every other team once, and will have exactly one idle round or "bye." If the number of teams, n, participating in the tournament is even then it is possible to complete the tournament in *n-1* rounds. Each round will consist of $\frac{n}{2}$ games, which each team participating in each round.

One approach to scheduling a tournament is the "polygon method." You can best explain the polygon method by going thorough an example. In this example we will schedule a tournament consisting of 5 teams (A, B, C, D, and E).

- Step 1: Draw the vertices of a regular *n*-gon. Each vertex represents a specific team.

```
        A
        o
  E o       o B

      o   o
     D     C
```

- Step 2: Draw $\frac{n-1}{2}$ line segments connecting the vertices such that the following conditions hold:
    - No vertex has more than one segment drawn to/from it.
    - No segment is a rotation or reflection of another segment.

- The easiest way to do this is to have the two bottom vertices across from each other and leave the top vertex point unmatched. This vertex represents the team that has a "bye."

```
        A
        o
  E o-------o B

      o---o
     D     C
```

- Each segment connecting two vertices represents two teams playing each other in the first round.

- Step 3: Rotate the polygon $\frac{1}{n}$th of a circle clockwise or one vertex point and repeat step 2.

```
        E
        o
  D o-------o A

      o---o
     C     B
```

- The new segments represent the match-ups for round two.

- Step 4: Continue rotating the polygon until you return to the original position. Each rotation represents the match-up for the corresponding round.

Round three:

```
     D
     o
C o-------o E

   o---o
   B   A
```

Round four:

```
     C
     o
B o-------o D

   o---o
   A   E
```

Round five:

```
     B
     o
A o-------o C

   o---o
   E   D
```

- The next rotation brings you back to the original position, so the schedule is complete.

- The final schedule looks like this:

Round	**Games**	**Idle**
1	E-B, D-C	A
2	D-A, C-B	E
3	C-E, B-A	D
4	B-D, A-E	C
5	A-C, E-D	B

The case involving an even number of teams is a slight variation of the odd case. First you must draw a *(n-1)*-gon and place a point in the center of the polygon.

```
        A
        o

  E o         o B

       o F

     o     o
     D     C
```

The remaining steps are the same, except the idle team now plays the team represented by the center point.

```
          A
          o
          |
E o-------+-------o B
          |
         o F

      o-------o
      D       C
```

Also when the polygon is rotated, the center point remains fixed.

```
          E
          o
          |
D o-------+-------o A
          |
         o F

      o-------o
      C       B
```

One resulting schedule for a 6-team round robin tournament is:

Round	**Games**
1	E-B, D-C, A-F
2	D-A, C-B, E-F
3	C-E, B-A, D-F
4	B-D, A-E, C-F
5	A-C, E-D, B-F

This works for a number of reasons. In the odd case of step 2, requiring $\frac{n-1}{2}$ segments ensures that there is only one team that receives a bye in each round. Therefore the minimum number of rounds results. The restriction that no vertex has more than one segment drawn to or from it guarantees that no team is scheduled for than one game in each round. The restriction that no segment is a rotation or reflection of another segment guarantees that no match-up is repeated.

In applications to games and sports, often times the schedule of play must satisfy other additional requirements. For example, home-away schedules can be represented using the polygon method by incorporating directed edges. There are a number of other schedule requirements that can complicate making a round robin tournament schedule: time constraint, field constraints, and teams that are unable to play each other for example.

After completing a the detailed explanation of the polygon approach to scheduling a tournament, the students can be put in groups and asked to create a schedule for their local high school team that incorporates any of the restraints that are appropriate. The teacher should guide the groups in discussion of how the additional requirement can be incorporated into the schedule making. Some of the requirements can be incorporated rather easily while others require some deeper and more critical thinking. If the students show continuing interest, further discussion of other types of scheduling can result: elimination tournaments, season schedules, etc.

References

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J. Dinitz, D. Stinson, Contemporary Design Theory, A Collection of Surveys, Wiley-Interscience, 1992

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