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On the Timing and Application of Pesticides*

Darwin C. Hall and Richard B. Norgaard

The model developed by J. C. Headley to illustrate the entomologists' concept of the economic threshold is presented and criticized. A two-variable model directed at the problems of optimal timing and pesticide application as well as optimal pest population level is presented.

For economic, ecological, and social reasons, preventative spraying with heavy doses of nonselective, persistent insecticides is becoming obsolete. Pest resistance, rapid pest resurgence due to a lack of natural control by predators recently killed from application of pesticides, socially unacceptable environmental costs, and other phenomena resulting from preventative spraying have gradually led to more remedial spraying with selective pesticides. Entomologists are now promoting integrated control, i.e., using the best combination of all known inputs and techniques including biological controls, cultural practices, and chemical approaches. The philosophy of integrated control is well established; but, due to the disaggregated approach of past entomological research and the limited attention paid by economists to the problem, methods are still ad hoc. The dichotomy between the state of the philosophy and that of the practice is illustrated by considerable concern over the "economic threshold," a term used by entomologists to denote the pest population level at which controls should be initiated [3, p. 240]. Entomologists advocate using all inputs in their best combination and simultaneously admit considerable uncertainty as to when and how even a single control input, such as an insecticide, should be used.

The Headley Model

The definition of the economic threshold has recently been investigated by J. C. Headley within the framework of a simple pest population growth model and a single application of a pesticide [2]. The model relates crop damage in dollars to pest density and time. The economic threshold is that population level where the marginal benefit from damage prevented by the control program is equal to the marginal cost of realizing that population through a control program. The model has four basic elements: a pest population growth function (equation 1.1), a pest damage function (equation 1.2), a product yield function (equation 1.3), and a pest control cost function (equation 1.4). Each of these elements is presented below.

(1.1) $$P_t = P_{t-n}(1+r)^n$$

(1.2) $$D_t = bP_t^2 - A$$

(1.3) $$Y = N - cD_t$$

(1.4) $$K = \frac{h}{P_{t-n}}$$

where

P_t = pest population level at time period t, the harvest time;

P_{t-n} = pest population level n periods prior to t;

r = net growth rate of the pest population per time period;

D_t = cumulative damage at time period t, given as a function of P_t, where the pest population has grown from P_{t-n} to P_t with no external interruption;

A = a constant related to the pest damage tolerance level based on the recuperating potential of the crop;

b = a parameter relating units of pest population to units of crop damage;

Y = realized product yield at harvest time (t) in dollars;

N = potential yield at harvest time if no pest damage, expressed in dollars;

c = a parameter relating damage units to dollars;

K = total cost in dollars of reducing the pest population to P_{t-n} at time $t-n$;

and

h = a parameter relating the inverse of population units to dollar units of control costs.

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Combining the first three equations yields:

$$(1.5)\quad Y = N - c\{b[P_{t-n}(1 + r)^n]^2 - A\}.$$

The marginal change in yield due to an incremental increase in the pest population level at time period $t-n$ is:

$$(1.6)\quad \frac{dY}{dP_{t-n}} = -2cb(1 + r)^{2n}P_{t-n}.$$

The marginal change in the cost of pest control due to an incremental change in pest population at time period $t-n$ is:

$$(1.7)\quad \frac{dK}{dP_{t-n}} = -\frac{h}{P_{t-n}^2}.$$

Equating marginal revenue to marginal cost determines the optimal level to which the population should be reduced during time period $t-n$:

$$(1.8)\quad P_{t-n} = \left(\frac{h}{2cb(1 + r)^{2n}}\right)^{1/3}.$$

This is Headley's economic threshold. It indicates the level to which the pest population should be reduced during time period $t-n$ such that the damage due to the growing pest population between $t-n$ and harvest time t is minimized subject to the cost of pest control.

Headley's model nicely illustrates several of the factors which must be considered in any definition of the economic threshold. Nevertheless, it has several limiting assumptions. First, Headley implicitly assumes that the period of pesticide application $t-n$ is "entomologically determined."[1] Thus pest damage prior to $t-n$ cannot be controlled and is not considered in his model. Figure 1 illustrates the relationships between pest population and time in the model. The economic threshold, as defined by Headley, is the level P_{t-n} to which the population should be reduced rather than the population level $P_{t-n-\Delta}$ at which controls would be initiated. Previous definitions of the economic threshold by entomologists have emphasized this latter population level. The vast majority of pests do not have a crucial stage when they are vulnerable to pesticides; hence, the question of when to apply pesticides must be addressed. Indeed,

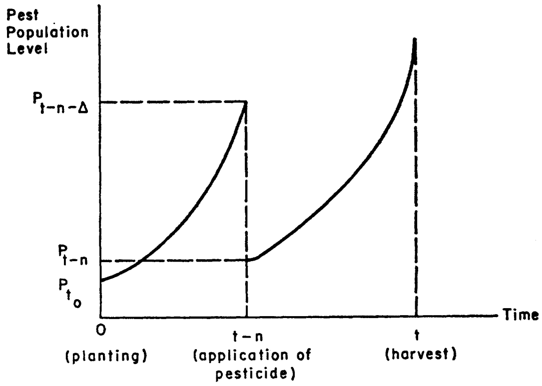

Figure 1. Pest population levels over time

the effectiveness of most pesticides is fairly well documented. The important issues are concerned with when they should be used.

Second, the cost of control must be a function of the level of population before controls are initiated, $P_{t-n-\Delta}$, and the difference between $P_{t-n-\Delta}$ and the level reached after control, P_{t-n}. Since $P_{t-n-\Delta}$ is a function of P_0, the pest population at the time of planting, Headley's cost function is valid only for a unique, unspecified value of P_0. It should be noted, furthermore, that, if the time of pesticide application $t-n$ is a variable, the cost function must be considerably respecified.

A Two-Variable Model

The following model corrects the above deficiencies yet retains as much as possible of the simple general form of Headley's model. It consists of five elements: a pest population growth function (equation 2.1), a pest population kill function (equation 2.2), a pest population damage function (equation 2.3), a product yield function (equation 2.4), and a pesticide cost function (equation 2.5). These are presented below.

Pest population growth and kill functions:

$$(2.1)\quad P(t) = \begin{cases} P_0e^{rt} & \text{for } t_0 \le t \le t_i \\ (P_0e^{rt_i} - K)e^{r(t-t_i)} & \text{for } t_i < t \le t_h \end{cases}$$

$$(2.2a)\quad K = K^*[X, P(t_i)] = K^*(X, P_0e^{rt_i})$$

$$(2.2b)\quad K = K'(X, t_i, P_0, r)$$

where

r = pest population growth,
t_0 = planting time,
t_i = pesticide application time,

[1] This assumption is not made clear in the original paper but is implicitly stated in a later letter from Headley to the authors dated February 3, 1972: "In fact, the population level P_{t-n} is entomologically determined as the *crucial state* of the insect" (*italics* ours).

t_h = harvest time,
K = pests killed by pesticide application,
P_0 = initial pest population at t_0,

and

X = quantity of pesticides.

The kill function indicates that the number of pests killed is a function of how much pesticide is applied and how many pests are present when the pesticide is applied. $P(t_i)$ is a function of t_i, r, and P_0; but, since the parameters P_0 and r are assumed to be fixed for any given planting period and pest, the abbreviation is as follows:

$$K = K(X, t_i). \tag{2.2c}$$

Pest population damage function:

$$d(t) = bP(t) \tag{2.3a}$$

$$D(t_2 - t_1) = \int_{t_1}^{t_2} d(t)dt \tag{2.3b}$$

where

$d(t)$ = instantaneous rate of crop damage in physical units due to pests (note that $d(t)$ is piecewise continuous over t_0 to t_h);
b = a parameter which specifies the rate of crop damage in physical units per pest;

and

$D(t_2 - t_1)$ = cumulative crop damage between time t_1 and time t_2.

Total damage at harvest time can be broken into two parts, one before pesticide application and one after:

$$D(t_h - t_0) = \int_{t_0}^{t_i} d(t)dt + \int_{t_i}^{t_h} d(t)dt = D_1 + D_2. \tag{2.3c}$$

$$D(t_h - t_0) = \frac{b}{r}\{(e^{rt_h} - e^{rt_i}) \cdot [P_0 - e^{-rt_i}K(X, t_i)] + P_0(e^{rt_i} - 1)\} \tag{2.3d}$$

when $t_0 = 0$. This is derived by substitution of equations (2.1), (2.2c), and (2.3a) into equation (2.3c).[2]

Product yield function:

$$Y = N - D(t_h) \tag{2.4}$$

where

Y = physical yield at harvest,
$t_0 = 0$,

and

N = physical yield if no damage occurs from pests.

Pesticide cost function:

$$C = \alpha X \tag{2.5}$$

where

C = total cost of pesticides,
X = number of units of pesticides,

and

α = cost of purchasing and applying a unit of pesticide.

Profit can now be written:

$$\pi = \beta Y - C = \beta[N - D_1 - D_2] - \alpha X. \tag{2.6a}$$

Substituting equation (2.3c) into (2.6a) results in the following:

$$\pi = \beta N - \frac{\beta b}{r}\{(e^{rt_h} - e^{rt_i}) \cdot [P_0 - e^{-rt_i}K(X, t_i)] + P_0(e^{rt_i} - 1)\} - \alpha X. \tag{2.6b}$$

The economic threshold is the population level $P(t_i)$ associated with the two decision variables, the optimum application time t_i, and the optimum quantity of pesticide X, which simultaneously maximize profits. Differentiating equation (2.6b) with respect to X and t_i results in the first-order conditions:

$$\pi_X = -\alpha + \frac{\beta b}{r}[e^{r(t_h - t_i)} - 1]K_X(X, t_i) = 0. \tag{2.7a}$$

[2] The pest population damage function may be rewritten as:

$$d(t) = \begin{cases} b[P(t) - A(t)] & \text{for } P(t) \geq A(t) \\ 0 & \text{for } P(t) \leq A(t) \end{cases}$$

where $A(t)$ is the pest damage tolerance level, the maximum pest population level which, at each point in time, results in no discernible loss due to pest damage at harvest time.

Two cases must be investigated. The first is whether P_0 is less than $A(t_o)$. The second is whether $P_o e^{rt_i} - K$ is less than $A(t_i)$. For simplicity, suppose A is constant. If $P_o < A$, there exists some t_* such that equation (2.3d) may be rewritten as $P_0 e^{rt^*} = A$ or $t_* = 1/r[ln(A) - ln(P_0)]$. Similarly, if $P_o e^{rt_i} - K$ is less than A, there exists some t_{**} such that $t_{**} = l/r[ln(A) - ln(P_o e^{rt_i} - K)] + t_i$.

Using equations (2.3) and (2.4), total damage can be broken into two periods consisting of damage before and after pesticide application: $D(t_h - t_o) = D[t_i - \max \cdot (t_*, t_o)] + D[t_h - \max(t_{**}, t_i)]$. The problem of applying the above equation is that the max (t_{**}, t_i) is not known *a priori*.

$$\text{(2.7b)} \quad \pi_{t_i} = [1 - e^{r(t_h - t_i)}] K_{t_i}(X, t_i) + r e^{r(t_h - t_i)} K(X, t_i) = 0.$$

Equations (2.7a) and (2.7b) indicate that the economic threshold is also a function of the time of harvest, the rate of pest population growth, the rate of damage to crops per pest, the effectiveness of the pesticide, the cost of pesticide, and the price of the crop. Rewriting equations (2.7a) and (2.7b) results in the following:

$$\text{(2.7c)} \quad K_X(X, t_i) = \frac{\alpha r}{\beta b[e^{r(t_h - t_i)} - 1]} \cdot$$

$$\text{(2.7d)} \quad K_{t_i}(X, t_i) = \frac{r K(X, t_i) e^{r(t_h - t_i)}}{e^{r(t_h - t_i)} - 1} \cdot$$

Since t_h is greater than t_i in both case 1, condition 2, and case 2, $e^{r(t_h - t_i)} - 1$ is positive. Therefore, since α, r, β, and b are also positive, $K_X(X, t_i)$ is positive, indicating that more pesticide indeed kills more pests. Since $K(X, t_i)$ is positive for positive values of X, $K_{t_i}(X, t_i)$ is positive. This is intuitively appealing since, for later t_i's, the pest population density increases, and a fixed quantity of pesticide would kill a larger number of pests when the pest density is greater. It is rather interesting to note that product and pesticide costs affect K_X but not K_{t_i}. This means that, if the quantity of pesticides to be applied is for some reason fixed, the timing of application would not be affected by changes in the prices of the pesticide or the product. In this case the only determinants of t_i are the effectiveness of the pesticide and the pest population growth rate. To solve explicitly for the economic threshold, the form of the pest population kill function must be specified.[3]

Conclusions

The foregoing model is an improvement on Headley's model in that both the timing and the quantity of pesticide applied are variables. Like Headley's model, it provides rigor to the definition of the concept of economic threshold but is too simple for practical application. In reality, the decision to spray is complicated by the presence of more than one pest and interrelationships between pests, beneficial predators, and parasites which may also be killed by the pesticide. Furthermore, the toxicity of some pesticides is persistent; weather, pest density, and the availability of food, among other factors, influence net population growth rates; plants are sometimes more and other times less sensitive to pest damage; biological control inputs can be introduced into the system; and the system itself can be manipulated to reduce pest damage. Recent work by Carlson has brought out the importance of uncertainty and the risk preferences of pest managers [1]. As these additional factors are introduced, mathematical models rapidly become unmanageable.[4] "Black box" approaches, which explicitly recognize the uncertainty of future events and the risk preferences of the decision-maker, will be necessary in the next generation of models.

[3] We note that profit maximization must be constrained to exclude negative pest populations and applications of negative quantities of pesticide. The second-order condition $\pi_{XX} < 0$ and the previous result $e^{r(t_h - t_i)} - 1 \geq 0$ imply that $K_{XX} < 0$. This means additional increments of pesticide must be less effective than previous increments. Summarizing, this model has a unique interior solution if the following three plausible assumptions are valid: the number of pests killed increases with an increase in the quantity of pesticide applied; the number of pests killed increases at a decreasing rate; and a given quantity of pesticide will kill more pests if the pest density increases.

[4] The mathematical complexity of the simple pest-predator relationship without the other factors discussed above has been explored recently by Paul A. Samuelson [4].

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