

CALIFORNIA STATE UNIVERSITY, LONG BEACH

# THE MATHEMATICS COLLOQUIUM

Presents

Professor Anthony Shaheen

Department of Mathematics  
California State University, Los Angeles

Speaking on

*Expander and Ramanujan Graphs*

**Friday, March 6, 2009**

**12:00noon – 1:00pm**

**FO3-200A**

Abstract:

Think of a communications network as a graph. Each vertex represents a person (or a telephone, or a computer) in the network; two vertices are connected by an edge if they can communicate directly with one another. When designing a communications network, one naturally wishes that any message originating somewhere within it will propagate as rapidly as possible. There are various measurements which describe how well a graph spreads messages. One of these is called the expansion constant of the graph. Roughly speaking, the better the expansion constant, the better the network.

A graph is called  $k$ -regular if every vertex has degree  $k$ . In a large communications network, one wants to minimize the number of edges used (we do this by making all our graphs  $k$ -regular and keeping  $k$  fixed). If one attempts to create  $k$ -regular graphs with more and more vertices, but without increasing the degree (keeping  $k$  fixed), the expansion constants of the graphs may very well become infinitesimally small as the networks become poorer and poorer. A family of expander graphs is an infinite sequence of  $k$ -regular graphs, with  $k$  fixed, such that the expansion constant does not go below a fixed lower bound.

One can also measure expansion of graphs using what is called the spectral gap of a graph, which is closely related to the expansion constant of a graph. Roughly speaking, graphs with the best possible spectral gap are called Ramanujan graphs. Families of Ramanujan graphs are also expander graphs (as long as  $k$  is larger than 2).

In this talk, I will give an introduction to this subject. The talk will be accessible to undergraduates. That is, as long as you know what an eigenvalue is, then you will be okay.