Department of Mathematics & Statistics

Program Review

Self-Study Report

June 2006

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Mission Statement

Mathematics and statistics are responses to the fundamental human endeavor to understand the world. Furthermore, modern technological society relies on developments in these disciplines. Consequently, the Department of Mathematics and Statistics at California State University, Long Beach, as in most universities, plays a vital and indispensable role in the lives of the students and faculty.

The Department's mission is two-fold: to assist in integrating students into society by educating them in these fields; secondly, to build and foster an excellent faculty, which contributes to the development of mathematics and statistics and serves as a resource of mathematical expertise for the people of this state. This department aims to fulfill this mission by performing the following major functions:

1. Provide superior General Education instruction for every student on campus.

2. Deliver quality instruction via a large collection of service courses to a significant variety of departments and majors.

3. Sustain a considerable and qualified group of undergraduate majors in the areas of general mathematics, applied mathematics, mathematics education and statistics.

4. Maintain vibrant graduate programs in the fields of general and applied mathematics, mathematics education and applied statistics.

5. Enrich the intellectual life of the campus and the general mathematical and statistical community by providing expertise and practicing scholarly activity.

In order to fulfill these functions the department should do the following:

¡ seek, constantly, ways to improve its teaching methods thereby deepening student understanding, including the use of new technology as a teaching tool, and maintain a diverse and modern curriculum at all levels;

¢ convey to all students a sense of the relevancy of mathematics and statistics, and of the importance of analytical and quantitative skills in contemporary society;

£ provide placement and advising tools for students so as to promote a high level of success among the many students taking its courses;

¤ make available intervention tools to maintain a high level of success among all its students;

¥ schedule appropriate classes at the appropriate times to try to meet the diverse needs of the various communities of students that attend CSULB;

¦ maintain communication with the departments whose majors are served by its courses in order to ensure that the courses are providing the knowledge and skills needed, within the constraints of usefulness to all students in those courses and of the maintenance of mathematical integrity;

§ foster an environment for its students in which the excitement and vivacity of mathematical and statistical activity is apparent, and in which students carry that enthusiasm back to their professional lives regardless of whether they are secondary school or community college teachers, applied mathematicians or statisticians in commerce and industry or graduate students pursuing higher degrees;

¨ impart appropriate training in mathematics and statistics for graduate students who will use that training professionally as mathematicians, statisticians or as teachers of mathematics while also providing adequate preparation for the students who will pursue doctoral studies;

© encourage faculty to remain active in their discipline by reading and learning new mathematics or statistics, attending and participating in conferences related to these subjects, sharing the results of successful teaching approaches, writing for appropriate journals or writing textbooks, or developing original mathematics or statistics or their applications.

Below we will address each of these individual points starting with

- seek, constantly, ways to improve its teaching methods thereby deepening student understanding, including the use of new technology as a teaching tool, and maintain a diverse and modern curriculum at all levels

The Department of Mathematics and Statistics is a large department, which amounts (in terms of FTESs) to approximately four ninths of the College of Natural Sciences and Mathematics as the following table attests:

Number of FTES's									
	F01	S02	F02	S03	F03	S04	F04	S05	F05
Below 100	416.6	262.0	239.4	123.2	220.6	113.4	263.8	156.8	312.2
100-level	1027.1	890.5	1027.0	832.8	1001.5	766.3	887.3	759.6	974.5
200-level	125.9	136.3	139.3	150.9	134.7	119.9	122.9	115.4	114.2
300-level	136.1	156.7	181.1	195.2	220.3	197.5	207.7	183.5	195.5
400-level	61.0	70.6	73.7	23.4	77.5	87.9	95.9	96.4	87.2
Graduate	15.9	12.0	16.7	16.4	18.5	17.4	26.9	21.7	37.2
Total	1782.7	1528.1	1677.1	1341.8	1673.1	1302.5	1604.7	1333.5	1720.8

When viewed in percentages, the previous table illustrates the salient fact that a large component of the Department's role is that of freshman instruction, that being in either a general education capacity or service components to different majors.

	F01	S02	F02	S03	F03	S04	F04	S05	F05
Below 100	23.4%	17.1%	14.3%	9.2%	13.2%	8.7%	16.4%	11.8%	18.1%
100-level	57.6%	58.3%	61.2%	62.1%	59.9%	58.8%	55.3%	57.0%	56.6%
200-level	7.1%	8.9%	8.3%	11.2%	8.0%	9.2%	7.7%	8.7%	6.6%
300-level	7.6%	10.3%	10.8%	14.5%	13.2%	15.2%	12.9%	13.8%	11.4%
400-level	3.4%	4.6%	4.4%	1.7%	4.6%	6.8%	6.0%	7.2%	5.1%
Graduate	0.9%	0.8%	1.0%	1.2%	1.1%	1.3%	1.7%	1.6%	2.2%
Total	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%	100.0%

There will be more said about the general education and service courses below. For the students with majors and minors in the Department, the programs that the Department supports are as follows:

Four options for the Bachelor of Science degree:

BS in Mathematics

- General (MATHBS01)
- Applied Mathematics (MATHBS02)
- Mathematics Education (MATHBS03)
- Statistics (MATHBS04)

Three possibilities for minors:

Minor in Mathematics (MATHUM01)
Minor in Applied Mathematics (MATHUM02)
Minor in Statistics (MATHUM03)

The single subject credential:

Single Subject Credential in Mathematics (165)

Four options for the Master's degree:

MS in Mathematics

General (MATHMS01)
Applied Mathematics (MATHMS02)
Applied Statistics (MATHMS03)
Mathematics Education for Secondary School Teachers (MATHMS04)

The curriculum for each of the programs is regularly examined versus the curriculum in other similar universities (see Appendix 1). Also in Appendix 2 is a list of the most important courses in each of the undergraduate options as well as their aims and goals.

The Department supports and staffs three computer labs, two equipped with PCs and one with Macs. These are extensively used for MATH 323, MTED 402 and 301, and sporadically used for other MATH and MTED classes. Of the general education classes, MATH 180 is taught with a heavy component of technology.

The faculty in the department regularly attends local and national meetings of professional organizations such as the Mathematical Association of America, the American Statistical Association, the American Mathematical Society, National Council of Teachers of Mathematics and the Society for Industrial and Applied Mathematics, so they are cognizant of national and local trends in curricular and professional matters.

¢ convey to all students a sense of the relevancy of mathematics and statistics, and of the importance of analytical and quantitative skills in contemporary society

In an era when more and more emphasis is being placed on the ability to process, manipulate, and convey information—especially quantitative information—the Department recognizes the importance of instilling in all students the ability to think analytically when required and make intelligent decisions when confronted with quantitative information.

We are not concerned here with those students for whom intense training in mathematics is required in their major, but with those students whose exposure to mathematics is generally limited to one or two lower-division courses.

The Department offers a variety of courses that satisfy the General Education requirement—as well as pre-baccalaureate courses (Math 1 and Math 10) as needed to prepare students to take a course at college level. We present here a brief description of 100-level courses that are taken by a sizeable number of students, how they help to fulfill the mission described above, and the student demographic that is most likely to take these courses.

Math 103 (Mathematical Ideas)—The vast majority of students who take this course are liberal arts majors, to whom math is frequently an anathema. The purpose of this course is twofold: to teach the students some interesting and practical topics, but also to help them appreciate, at least to some extent, the beauty and utility of mathematics. This is perhaps the only 100-level course whose content is, to some extent, at the instructor's option—but the departmental syllabus (which must be followed by all part-time instructors) includes an introduction to the mathematics of finance, and a section on probability and statistics. These topics enhance the student's ability to deal with the quantitative analyses that must be made in daily life.

Math 112 (College Algebra)—Although this course is occasionally selected to satisfy the General Education requirement by liberal arts majors who are relatively comfortable with their ability to perform algebraic manipulations, it is more often chosen by majors in fields that either require it (such as life or social sciences) or by students who are tentatively considering majors in more mathematically intensive subjects, but feel that their algebraic skills need strengthening. The course content includes significant components in algebraic reasoning and problem analysis.

Below some proposed changes to 112 are outlined.

Math 114 (Finite Mathematics) and Math 115 (Business Calculus) – These two courses are grouped together because they are required by the business school for all their majors. Math 114 emphasizes probability and statistics in making successful decisions in business situations, and Math 115 covers a broad variety of business-related problems which calculus is uniquely adept at analyzing. Although the vast majority of the students who take this course are business majors, there is some enrollment from students who have sufficient confidence in their mathematical skills that they are comfortable taking more challenging courses than Math 103 and Math 112. In addition, the Department regularly offers sections of these courses for the University Honors Program.

Math 180 (Statistics for Everyday Life) – This course is usually taken by students in a variety of majors including those that traditionally require a substantial background in statistical analysis. The course is designed to give students the key statistical tools that are used in analyzing data, as well as the ability to recognize when statistical information is being used erroneously.

In Appendix 3 one can find the detailed descriptors of the courses with their aims and goals together with the outlined changes to 112 that are described below.

£ provide placement and advising tools for students so as to promote a high level of success among the many students taking its courses;

In addition to the CSU system-required ELM (Entry Level Mathematics) Exam, the Department has started requiring the MDPT (Mathematics Diagnostic Placement Test) from those students interested in enrolling in either regular calculus (Math 122) or Biological Sciences Calculus (Math 119A). The Department is just beginning to gather information on the efficacy of this effort.

The department maintains an extensive presence at the advising workshops for incoming students run by the SOAR program. At least one faculty member (usually Dr. Merryfield) and at least one staff member are normally present at each workshop. The staff member is there to facilitate enrollment. Most incoming mathematics majors speak to a faculty member when they enter.

Continuing students declaring a mathematics major must speak to an advisor in order to make that declaration; the advisors use that conversation to outline the requirements and estimate the time needed to finish. The same applies to students declaring minors.

We rely on the normal procedures of university life to bring students in to see advisors, all of whom maintain frequent office hours and are among the most accessible faculty members. Such procedures include a mandatory consultation for second semester freshmen, seeking permission for such things as taking a course credit/no credit, or filing for graduations. Some special programs mandate consultation and schedule approval every semester, but most students are not in such programs. We encourage students to consult an advisor more often than that, and many do, but we do not force or require such visits.

¤ make available intervention tools to maintain a high level of success among all its students;

The Department supports through lottery funds the Mathematics Tutoring Center which is open suitable hours throughout the year. Its purpose is to assist students who have question in their courses. The Department also sponsors a center to tutor the Mathematics Education multiple subject majors that may need assistance. The last academic year there was an additional tutoring center strictly for students in Math 1 and Math 10, the pre-baccalaureate courses.

The fact is that the Department has a large remedial component each semester (1000 students), and one of the difficulties is that while some of the students in these pre-baccalaureate courses will eventually take a calculus-based course, the vast majority of them will not. Thus Math 10 had to prepare a student in a sufficiently demanding fashion so as to eventually be able to handle the rigors of calculus, yet, however, that level of rigor was perceived by most students as extreme and unnecessary.

Thus starting in the Fall of 2007, the Department will offer two pre-baccalaureate courses in intermediate algebra: Math 7 (Basic) and Math 11 (Enhanced). The former course will entitle the student to take only GE courses that are not calculus-based. Thus, the Department is adding to the aforementioned Math 103, Math 114 and Math 180, Math 101 (Pre-calculus Trigonometry) and Math 109 (Modeling with Algebra) as possibilities for the students that take Math 7. While Math 112 will eventually be done away with, the department will add to its curriculum Math 113 (Pre-Calculus Algebra). One can find descriptions of these in Appendix 3.

¥ schedule appropriate classes at the appropriate times to try to meet the diverse needs of the various communities of students that attend CSULB

The Department prides itself in always making sure enough sections and at the appropriate times for necessary courses are offered so as to allow all students flexibility. The reader is referred to Appendix 4 for evidence in support of this from throughout the campus.

¦ maintain communication with the departments whose majors are served by its courses in order to ensure that the courses are providing the knowledge and skills needed, within the constraints of usefulness to all students in those courses and of the maintenance of mathematical integrity.

The Department has established a position named Lower-Division Service Course Coordinator. The duties of this faculty member include in-depth and continuous communications with various departments on campus so at to assure the curriculum of in-service courses to those departments are aligned with their wants and needs. Other duties include selection of text-book for all courses not taught by tenured or tenured-track personnel as well as the syllabi of those courses.

§ foster an environment for its students in which the excitement and vivacity of mathematical and statistical activity is apparent, and in which students carry that enthusiasm back to their professional lives regardless of whether they are secondary school or community college teachers, applied mathematicians or statisticians in commerce and industry or graduate students pursuing higher degrees;

Undergraduate advising (for majors, minors, and other undergraduates interested in mathematics courses) is primarily the responsibility of the Undergraduate Associate Chair Dr. Kent Merryfield, and the Undergraduate Advisor, Dr. Will Murray. They both receive assigned time for their efforts. Other members in the department also contribute to advising.

For post-baccalaureate students in the Single Subject Teaching Credential program, we have a Credential Advisor, who has been Dr. Angelo Segalla. The Credential Advisor receives assigned time support. For advanced undergraduates intending to pursue teaching careers, the responsibilities of the Credential Advisor and the Undergraduate Advisor overlap somewhat, and Drs. Segalla, Murray, and Merryfield maintain close contact and frequently discuss particular students.

Three years ago, the department instituted for undergraduates an Honors in the Major program which requires extra units and an undergraduate thesis. Since then, about two students per year have taken advantage of this program. Most of these students have gone on to Ph.D. programs in mathematics. We plan to maintain this at approximately its current level, encouraging our best student to enter this program. No monetary support is involved.

In addition, top students with a flair for problem-solving are encouraged to take MATH 491 (Honors Seminar in Problem Solving) each fall and to participate in the Putnam Competition. Over the last six years, CSULB has rank about 110th or 120th out of more than 300 schools on five different and placed three students among the top 500 individuals in the country.

A survey was taken of our graduate and undergraduates students to attempt to measure their level of satisfaction with the Departments offerings in advising, courses and instruction. The reader is encouraged to consult Appendix 5 for the positive results.

Finally, the Math Stat Student Association is quite active. The Association has organized and supported talks from former students about their job experiences as well as academic talks from professors and graduate students about the mathematical-statistical experience.

¨ impart appropriate training in mathematics and statistics for graduate students who will use that training professionally as mathematicians, statisticians or as teachers of mathematics while also providing adequate preparation for the students who will pursue doctoral studies;

The graduate curriculum in mathematics is designed to prepare students for careers as professional mathematicians, statisticians, or teachers of mathematics, as well as prepare them for further graduate study at the doctoral level. Although we have not kept complete data on the employment status of our graduates, anecdotal evidence suggests that nearly all are successfully employed or have completed doctoral degrees in these areas. We do, however, have complete data on the graduates in the Option in Applied Statistics, and these are representative of the other options. In the short three-year life of this option there have been 27 graduates; of these 20 are employed in various industries (financial 3, government 2, aerospace 3, pharmaceutical 2, medical 1, insurance 4, market research 1, other fields 3), 2 are teaching at local community colleges, 3 are pursuing a PhD in Statistics, 1 is teaching in high school, and the remaining 2 have part-time employment. These data attest to the success of the program in preparing students for professional employment, teaching, or doctoral studies.

The core course requirements in each of the graduate options in mathematics involve both breadth and depth requirements. The depth requirement prepares students for further graduate study. The breadth requirement gives students an advanced viewpoint for many of the mathematics topics taught in community colleges, thus preparing them to be knowledgeable and versatile teachers. In addition, graduate students employed as Teaching Associates gain valuable experience for future teaching careers at community colleges. The Option in Applied Mathematics and the Option in Applied Statistics prepare students for jobs in industry and government as well.

Specifically, students in the Master of Science in Mathematics are required to complete at least 3 of the core graduate courses (a breadth requirement), including at least two full-year sequences (a depth requirement). The core graduate courses are Abstract Algebra, Topology, Real Analysis, and Complex Analysis. These courses represent the fields of mathematics students are expected to know for comprehensive exams at the doctoral level. Students in the option typically take all the core courses as well as the elective courses Elliptic Curves and Functional Analysis.

Students in the Option in Applied Mathematics are required to take several analysis courses (including the core course Applied Analysis) intended to prepare them for further graduate study in applied mathematics. The applied courses, which prepare students for jobs in industry, include Numerical Analysis, Applied Nonlinear Ordinary Differential Equations, Partial Differential Equations, and Stochastic Calculus and Applications.

Students in the Option in Applied Statistics choose courses in modern and highly sought after applications of statistics in industry. The courses include Experimental Design and Analysis, Actuarial Science, Survey Sampling, Time Series, Statistical Quality Control, Data Mining, and Statistical Simulation. The Statistical Inference course, as well as other required mathematics courses, prepares students for further graduate study in statistics.

The Option in Mathematics Education for Secondary School Teachers is designed to enhance the effectiveness of secondary school teachers. The courses give students an advanced perspective on the teaching and learning of mathematics at the secondary school level. Specifically these courses include Algebra in the Secondary School Curriculum, Geometry and Measurement in the Secondary School Curriculum, Analysis in the Secondary School Curriculum, and Probability and Statistics in the Secondary School Curriculum.

Lastly,

© encourage faculty to remain active in their discipline by reading and learning new mathematics or statistics, attending and participating in conferences related to these subjects, sharing the results of successful teaching approaches, writing for appropriate journals or writing textbooks, or developing original mathematics or statistics or their applications.

The Department has a broad range of young and experienced faculty in a wide variety of fields in mathematics and statistics. Appendix 6 has a list of the faculty as well as their areas of expertise. Many members of the faculty have active research programs—in fact, all, but a handful, have a refereed publication in the last five years. The Department has encouraged and supported research grant solicitation from all its faculty.

The Department also supports The Friday Colloquia, given mostly by visitors from other universities, local, national or international.

The College of Natural Sciences and Mathematics has been giving 6 units of assigned time to faculty during their first two years of their tenure. The Department has occasionally given assigned time to faculty for research projects. More systemically, the Executive Committee has passed the following resolution:

> Untenured faculty during the 3rd and 4th years of their tenure-track appointment, in order to assist them to have an active research program, shall receive three units of assigned time per semester.

Similar resolutions for the 5th and 6th year will be addressed next year and then voted by the department. The vote is expected to be affirmative.

Naturally, the question of resources in order to provide for this assigned time will have to be addressed each semester.

Appendix 1

Comparison of Courses with other CSU's and mostly local Universities

	CSU								Private	
	Fullerton	SDSU	CSUN	Fresno			Millersvill,PA		Loyola	USC
103	2	2	2	2			2		1	0
112	2	0	2	0			2		1	0
114	1	0	0	0			9		1	1
115	2	2	2	0			2		2	1
117	2	2	2	2			2		2	2
119A	1.5	2	0	2			1.5 (=115)		2	1
122	2	2	2	2			2		2	2
123	2	2	2	2			2		2	2
180	2	2	2	2			1.5		2	0
MTED 110	1	1.5	2	2					1	0
233	2	2	2	2			2		1.5	1
247	1 (w. DEQ)	2	2	2			2 (recom 233)		1.5 (req 233)	1
341	1 (senior level)	2	1	2 (4 units)			2		0	2
355	1 (Prereq 347)	1 (3 classes)	1	1.5			1(2 classes)		1	1
361A	2	2	2	2			1.5 (1sem)		2	2
364A	1.5 (1.5 sem.)	1.5 (1sem.^)	1	1.5 (^)			1.5 (^)		1.5 (^)	1
380	1.5	2	2	2			2		1.5 (1 sem.)	1
381	1.5 (two classes)	2	2	2			2		0	2
310 / 410	1.5	0	0	1 (4 units, 1sem)			1 (1sem)		0	1
MTED 411	1	1	0	1					1	0
444	1 (2 sem.**)	1 (**)	1 (**)	1.5 (4 units)			1.5		1 (**)	1.5
470	2	2	2	2			2		0	0
480	0	2	2	0			0		0	0

Levels: **2:** Course fits very well
1: Course seems to exist but the fit may be far from perfect
0: Course does not seem to exist

Notes. 1. The list includes schools around here that have semester calendar plus one other (somewhat randomly chosen) school: Millersville University (located at Central Pennsylvania and has 7000 (resp. 1200) undergraduate (resp. graduate) students with a Master program in Math.).

2. It is more difficult to classify the MTED classes. Other schools have more general / generic titles like "Math/Geometry for teachers".

Comments:

1. What is unique about us?
   a. We require NumberTheory / Linear Algebra as a prerequisite for Abstract Algebra.
   b. We have two-semester sequences of Differential Equations and History of Math.

2. What is special about Fullerton?
   a. Each student is required to complete one of the (4) concentrations (18-20 units) and one of the (9) cognates (9-11 units).
   b. Their general Math requires (with equivalent course numbers):
      Algebra I, 361A, Topology and
      three of the following: Algebra II, 461, 451, 341 and Combinatorics.

Appendix 2
Undergraduate Courses for the Programs

In this appendix we include descriptors for what are considered some of the most crucial courses for the Majors in the Department.

The list of courses is

	General Option	Applied Math	Math Education	Statistics
Math 122	X	X	X	X
Math 123	X	X	X	X
Math 233	X	U	X	U
Math 247	X	X	X	X
Math 361A	X	X	V	X
Math 364A	V	X	V	U
Math 380	X	X	X	X
Math 381	U	U	V	X
Math 410	U	U	X	U
Math 444	X	U	V	U
Math 470	U	X	U	U

X Required and considered essential
V Required
U Not Required

MATH 122 CALCULUS I

Goals:

1. Students will learn, understand, and use the fundamental concepts of calculus, especially the derivative and the integral.

2. Students will use calculus to solve problems.

3. Students will read, understand, and use mathematical reasoning and symbolism, and employ them to communicate their ideas in a logical organized manner.

4. Students will examine applications of calculus to science and engineering and learn to model problems in these areas as calculus problems.

Assessment Method: Embedded questions throughout homework, quizzes, and exams. Discourse in class and office hours.

Illustration #1: To enable the Starship Enterprise to escape imminent peril from the Romulans, Spock must construct a space-time disrupter in the shape of a box with a volume of $120\,m^3$. The bottom of the box must have a length which is 3 times its width and must be made of dilithium crystal. The top of the box must be made of Kryptonite and the sides of the box must be made of compressed tribble hide. Spock will minimize the cost of the disrupter because Starfleet Command has been issued a prime directive from the Federation to balance its budget. Sulu reports that dilithium crystal costs 15 bars of latinum per square meter, Kryptonite costs 5 bars of latinum per square meter, and compressed tribble hide costs 3 bars of latinum per square meter. Find the cost of the disrupter that Spock constructs.

The student begins by translating the given information into mathematical form. Letting x be the width of the bottom of the box in meters, the length is calculated to be $3x$. Letting y be the height of the box in meters and V be the volume in cubic meters, the student uses the given volume of $120\,m^3$ and the geometrical formula for volume of a box as length times width times height to obtain

$V = (3x)(x)(y) = 3x^2y = 120$.[2,3,4] Now using C to denote the cost of the box in bars of latinum, the student recognizes the mathematical problem is to minimize C. The student sums the products of the areas of the sides of the box times the cost per unit area of the material used to obtain

$C = 15(3x^2) + 5(3x^2) + 3(3xy + 3xy + xy + xy)$
$= 60x^2 + 24xy$.[2,3,4] Using the equation for the volume of the box to solve for y in terms of x, the student finds that $y = 40x^{-2}$ and substitutes into the equation for cost to obtain a function of one variable $C(x) = 60x^2 + 24x(40x^{-2}) = 60x^2 + 960x^{-1}$.[2] The derivative of $C(x)$ is calculated to be $C'(x) = 120x - 960x^{-2}$ when $x \neq 0$.[1,2]

Since the width of the box must be a positive number and $C(x)$ is differentiable for all positive x, theorems from calculus imply that the desired maximum of $C(x)$ will occur at the critical point where $C'(x) = 0$ if there is only one critical point and $C''(x) > 0$ at that critical point.[1] Solving $120x - 960x^{-2} = 0$, the student finds that $x^3 = \frac{960}{120} = 8$, so $x = 2$ is the only critical point. Computing $C''(x) = 120 + 1920x^{-3}$, the student determines that $C''(2) = 360 > 0$ and hence the desired minimum occurs at $x = 2$.[1,2,3] Thus the student concludes that the cost of the disrupter that Spock constructs is $C(2) = 60(2)^2 + 960(2)^{-1} = 920$ bars of latinum.[2,4]

Illustration #2: Temperature during one winter day in Trantor is predicted to be $f(t) = (4e^{-t} - 8)\cos(2t + e^{-t})$ °C at t hours after midnight. If this prediction comes true, what would be the average temperature during that day?

Recognizing that the average value of $f(t)$ over the interval $0 \le t \le 24$ is asked for, the student uses the definition of average value of a function to set up the definite integral

$$f_{ave} = \frac{1}{24-0}\int_0^{24} f(t)\,dt = \frac{1}{24}\int_0^{24} (4e^{-t} - 8)\cos(2t + e^{-t})\,dt \;.^{1,2,3,4}$$

To evaluate the integral, the student attempts to use the substitution $u = 2t + e^{-t}$. Computing $du = 2 - e^{-t}\,dt$, the student finds that $-4du = (4e^{-t} - 8)\,dt$. Since $u(0) = 1$ and $u(24) = 48 + e^{-24}$, the integral becomes $f_{ave} = \frac{1}{24}\int_1^{48+e^{-24}} -4\cos u\,du$.[2,3] Using the Fundamental

Theorem of Calculus, the student concludes that the average temperature is

$$f_{ave} = -\frac{1}{6}\, sin\, u\,|_{1}^{48+e^{-24}} = -\frac{1}{6}\, sin\left(48 + e^{-24}\right) + \frac{1}{6}\, sin\, 1 \text{ °C} .^{1,2,4}$$

MATH 123 CALCULUS II

Goals:

1. Students will learn, understand, and use the fundamental concepts of calculus, especially the integral and infinite series.

2. Students will use calculus to solve problems.

3. Students will read, understand, and use mathematical reasoning and symbolism, and employ them to communicate their ideas in a logical organized manner.

4. Students will examine applications of calculus to science and engineering and learn to model problems in these areas as calculus problems.

Assessment Method: Embedded questions throughout homework, quizzes, and exams. Discourse in class and office hours.

Illustration # 1:

After a fierce attack by Empire stormtroopers, Princess Leia and Han Solo are left dangling at the end of a 50 ft long trilithium cable hanging from a walkway over the power pit of the Empire Deathstar. Chewbacca then pulls the cable all the way up so that they are all at the walkway. If the cable weighs 8 lb/ft, and Leia and Han together weigh 300 lb, find the work done by Chewbacca.

The student immediately observes that the work required to lift Leia and Han is just the force times the distance $= (300\,\text{lb})\ (50\,\text{ft}) = 15,000\,\text{ft-lb}$, so the problem reduces to finding the work required to lift the cable.[4] Since each cross-section of cable is at a different distance from the walkway, the student seeks to approximate the work by dividing the cable into n parts, each having length $\Delta x = \dfrac{50}{n}\,ft$. Numbering these parts 1 through n , a point on each part i is selected to represent the whole part in that its distance, x_i^*, from the walkway is used for all points in part i in the computation of approximate work.[3] The force of gravity on any part of the cable of length Δx is $(\Delta x\, ft)\,(8\, lb/ft) = 8\Delta x\, lb$, so the approximate work done on that part is $(8\,\Delta x\, lb)\,(x_i^*\, ft) = 8x_i^*\,\Delta x\, ft - lb$. Adding up the approximate work for all the parts, the student arrives at a total approximate work given by $\sum_{i=1}^{n} 8x_i^* \,\Delta x$.[3] The student observes that the larger the number of parts n used in the approximation, the better the approximation should be. Indeed, the exact work in lifting the cable should be $\lim_{n\to\infty} \sum_{i=1}^{n} 8x_i^* \,\Delta x$. Now the student recognizes that, according to the definition of definite integral, the desired limit is $\int_0^{50} 8x\,dx$.[1,3] Using the Fundamental Theorem of Calculus, the student calculates

$$\int_0^{50} 8x\,dx \;=\; 4x^2\,|_0^{50} \;=\; 4(50)^2 \;=\; 10,000\ ft - lb\ .$$ [1,2]

The student concludes that the work done by Chewbacca in lifting Leia, Han, and the cable is $15,000 + 10,000 = 25,000\, ft - lb$.[2,4]

Illustration #2: Find the Taylor series for $f(x) = e^x$ about the point $a = 0$. Use it to find a polynomial of degree 6 that approximates $g(x) = e^{-\frac{x^3}{2}}$ near 0. Find an upper bound on the absolute error if the polynomial is used to approximate $g(x)$ for values of x in $[-1, 1]$.

The student seeks to use the definition of Taylor series of a function.[1] Noting that $f^{(n)}(x) = e^x$ for all n , the student finds that

$$e^x \;=\; f(x) \;=\; \sum_{n=0}^{\infty} \frac{f^{(n)}(a)\,(x-a)^n}{n!} \;=\; \sum_{n=0}^{\infty} \frac{e^0\,(x-0)^n}{n!} \;=\; \sum_{n=0}^{\infty} x^n\ .$$ [3]

Replacing x by $2x^3$, the student arrives at

$$g(x) = e^{-\frac{x^3}{2}} \;=\; \sum_{n=0}^{\infty} \left(-\frac{x^3}{2} \right)^n \;=\; \sum_{n=0}^{\infty} (-1)^n\, \frac{x^{3n}}{2^n}$$

and uses the first three terms of this series to obtain $1 - \frac{1}{2}\,x^3 + \frac{1}{4}\,x^6$, a polynomial of degree 6 that approximates $g(x)$.[1,2,3] To determine an upper bound on E , the error when the polynomial is used to approximate $g(x)$ in $[-1, 1]$, the student observes that the above series for $g(x)$ is an alternating series.[1] The student then notes that, for $x \in [-1, 1]$, $\left|\frac{x^{3n+3}}{2^{n+1}}\right| < \left|\frac{x^{3n}}{2^n}\right|$ for all n and $\lim_{n\to\infty} \frac{x^{3n}}{2^n} = 0$ (by the Squeeze Theorem as $-\frac{1}{2^n} \le \frac{x^{3n}}{2^n} \le \frac{1}{2^n}$ and $\lim_{n\to\infty} \frac{1}{2^n} = 0$) , so the Alternating Series Estimation Theorem yields $|E| \le \left|\frac{x^9}{2^3}\right| \le \frac{1}{8}$.[1,2,3] The student concludes that the absolute error must never exceed $\frac{1}{8}$ for values of x in $[-1, 1]$.

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Major Goals:

The main goal of this course is to prepare students for higher mathematics. This is done by engaging students in deep problem-solving situations and techniques of proof that presage higher level topics. Through example and exercise, students will raise their general mathematical sophistication—the ability to read and write complex and convincing arguments. The mathematical reasoning in this course is practiced on fundamental topics that are foundational for higher mathematics. These topics include numbers, sets, induction, relations, functions, and counting techniques. The specific goals are as follows. Students will demonstrate the ability to

¬ Transform intuition into proof, and to differentiate between proof and opinion/example.

- Use the propositional and predicate logic and the language of sets, relations, and functions in writing mathematical proofs.

® Read and construct valid mathematical arguments (proofs), including proofs by induction, direct and indirect reasoning, proof by contradiction, and disproof by counterexample.

¯ Solve counting problems by applying the multiplication principle, the inclusion-exclusion principle, the pigeonhole principle, recurrence relations, and, in particular; the use of permutations and combinations.

° Use counting techniques to compute probabilities of events.

Representative Textbooks
A transition to advanced mathematics, by D. Smith, M. Eggen and R. St. Andre
A concise introduction to pure mathematics, by M. Liebeck

Assessment Method: Embedded questions throughout homework, quizzes and exams.

Illustration #1: Counting
Putting "Balls-into-Buckets" is a model for the following mathematical constructs.

1. Partitions: Modeled by distinguishable balls into identical buckets

2. Functions: Modeled by distinguishable balls into distinguishable buckets.
    a. Injective functions: at most one ball in each bucket
    b. Surjective functions: no empty buckets
    c. Bijective functions: exactly one ball in each bucket

3. The number of nonnegative solutions of a Diophantine equation can be modeled by identical balls into distinguishable buckets. For example, the number of solutions of x+y+z=10 is modeled by ten identical balls into three distinguished buckets.

Illustration #2: Relations

1. Define the relation on the set $\{1,-1,2,-2,3,-3\}$ by $a<b$ if a divides b. Is this relation a partial order? Prove your answer.
2. Define $a \approx b$ if ab is a square number. Is this relation an equivalence relation? Prove your answer.

Illustration #3: Functions

1. If $f \circ g$ is injective, prove that g is injective.
2. Give an example where $f \circ g$ is injective but f is not injective.

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Goals:

¬ Students will understand matrices as collectors of information, execute key operations involving matrices, and use these operations to solve relevant problems.

\- Students will be able to find all solutions to arbitrary systems of linear equations, by using Gaussian Elimination. The student will understand the process behind the reduction, and eventually understand the relation between solvability, null spaces and column spaces.

® Students will increase their level of mathematical linguistic ability by being able to deduce simple truths or give counterexamples to falsities involving the many concepts introduced in the course such as linear independence, spanning sets, dimension and basis.

¯ Students will increase their sense of mathematical rigor by being able to argue simple theorems concerning matrices and vectors as well as follow more sophisticated arguments.

° Students will be able to compute eigenvalues and eigenvectors of a matrix, and their relation to whether a matrix can be similar to a diagonal matrix or not.

± Students will be able to orthogonally diagonalize any symmetric matrix by using the Gram-Schmidt process to find an orthonormal basis for such a matrix.

Assessment Method: Embedded questions throughout homework, quizzes and exams.

Illustration #1: Of the three options on the right in the table, check **all** those that are applicable as to the possible number of solutions to the system $\mathbf{Ax} = \mathbf{b}$, assuming all the information that you are given. For some, only one option may apply, for others, two may apply, and still others all three may apply. For example, if you are told that $\mathbf{A}$ is 3×3, and that it has rank 3, then you should put a check only on the Unique column, since such a system will have a unique solution. On the other hand, if all you are given is that $\mathbf{A}$ is 3×3, then you should put a check in all three columns No solution, Unique Solution, and Infinitely many since all of these are possible under the assumptions.

#	$\mathbf{Ax} = \mathbf{b}$	No Solution	Unique Solution	Infinitely many solutions
1	$\mathbf{A}$ is 12×15 and $\mathbf{b} = \mathbf{0}$			
2	$\mathbf{A}$ is 12×15			
3	$\mathbf{A}$ is 15×10			

4	$\mathbf{A}$ is 12×12			
5	$\mathbf{A}$ is 15×10 and $\mathbf{b}=\mathbf{0}$			
6	$\mathbf{A}$ is 12×12 and $\mathbf{A}$ has rank as large as possible			
7	$\mathbf{A}$ is 12×15 and $\mathbf{A}$ has rank as large as possible			
8	$\mathbf{A}$ is 15×10 and $\mathbf{A}$ has rank as large as possible			
9	$\mathbf{A}$ is 15×10 and both $\mathbf{A}$ and $(\mathbf{A} \quad \mathbf{b})$ have rank as large as possible			
10	$\mathbf{A}$ is 12×15 and both $\mathbf{A}$ and $(\mathbf{A} \quad \mathbf{b})$ have rank as large as possible			

The student is expected to fill the table correctly - , ®, ¯.

In a similar fashion the student is expected to answer the following questions:

Illustration #2: Consider the veracity or falsehood of each of the following statements, and argue for those that you believe are true while providing a counterexample for those that you believe are false. Let $S=\{\mathbf{u}_1,\mathbf{u}_2,\mathbf{u}_3,\mathbf{u}_4\}$, $\mathbf{A}=(\mathbf{u}_1 \quad \mathbf{u}_2 \quad \mathbf{u}_3 \quad \mathbf{u}_4)$, $T=S\cup\{\mathbf{v}\}=\{\mathbf{v},\mathbf{u}_1,\mathbf{u}_2,\mathbf{u}_3,\mathbf{u}_4\}$ ®, ¯.

¡ $\mathbf{v}$ is in the span of T.

¢ $\mathbf{0}$ is in the span of T and in the span of S.

£ $\mathbf{Ax}=\mathbf{v}$ has a solution if and only if the span of S is the same as the span of T.

¤ The rank of $\mathbf{A}$ is 4 if and only if S is a linearly independent set.

¥ If T is linearly independent, then so is S.

Illustration #3 Show that the collection of symmetric matrices of size n is a subspace of the space of all square matrices of size n, and compute its dimension.

The proof should follow from elementary properties of the transpose. That $(a\mathbf{A}+\mathbf{B})^{\mathrm{T}}=a\mathbf{A}^{\mathrm{T}}+\mathbf{B}^{\mathrm{T}}$ for any scalar a and any matrices $\mathbf{A},\mathbf{B}$ implies that the collection of symmetric matrices ($\mathbf{A}^{\mathrm{T}}=\mathbf{A}$) is a subspace ®, ¯. To compute the dimension, one needs to find a basis. The student could start by doing the 2×2 case and obtain easily the basis $\begin{pmatrix}1&0\\0&0\end{pmatrix}$, $\begin{pmatrix}0&1\\1&0\end{pmatrix}$ and $\begin{pmatrix}0&0\\0&1\end{pmatrix}$. So it is 3-dimensional.

For the case $n=3$, an arbitrary symmetric case is $\begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix}$, so one can see that a basis will consist of 6 elements: $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ and $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. From there the student is expected to conclude that the basis will always have for arbitrary n, $\frac{n(n+1)}{2}$ elements so the dimension given by this number ¯.

Illustration #4: Suppose $\mathbf{A}$ is 10×10. Suppose $\mathbf{A}-\mathbf{I}$ has rank 3, $\det(\mathbf{A}+\mathbf{I})=0$ and $\det(\mathbf{A}-2\mathbf{I})=0$. Suppose the trace of $\mathbf{A}$ is 9. Decide whether $\mathbf{A}$ is invertible and whether $\mathbf{A}$ is similar to a diagonal matrix.

Here the student is expected to deduce that 1 is an eigenvalue with multiplicity at least 7 ®, °, and that also -1 and 2 are eigenvalues. Finally since the trace is 9, the last remaining eigenvalue is also 1. From there the student concludes that $\mathbf{A}$, while invertible, is not similar to a diagonal matrix ®, ¯, °.

Illustration #5 Let $\mathbf{A}=\mathbf{J}_5=\begin{matrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{matrix}$. To find an orthogonal matrix whose columns are eigenvectors, first one must compute the eigenvalues to be $0,0,0,0,5$ °. First one must find a basis for the null space of $\mathbf{A}$ - , ® such as $\mathbf{u}_1=\begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{u}_2=\begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$, $\mathbf{u}_3=\begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ and $\mathbf{u}_4=\begin{pmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$, and the observe that $\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ is an eigenvector for 5. Then using Gram-Schmidt, one gets $\mathbf{P}=\begin{pmatrix} \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{12}} & \frac{-1}{\sqrt{20}} & \frac{1}{\sqrt{5}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{12}} & \frac{-1}{\sqrt{20}} & \frac{1}{\sqrt{5}} \\ 0 & \frac{2}{\sqrt{6}} & \frac{-1}{\sqrt{12}} & \frac{-1}{\sqrt{20}} & \frac{1}{\sqrt{5}} \\ 0 & 0 & \frac{3}{\sqrt{12}} & \frac{-1}{\sqrt{20}} & \frac{1}{\sqrt{5}} \\ 0 & 0 & 0 & \frac{4}{\sqrt{20}} & \frac{1}{\sqrt{5}} \end{pmatrix}$, which satisfies: $\mathbf{P}^{-1}\mathbf{A}\mathbf{P}=\mathbf{D}$, the diagonal matrix with $0,0,0,0,5$ on the main diagonal ±.

Math 361A
Introduction to Mathematical Analysis

Goals:

1. Students will read and understand valid mathematical arguments.
2. Students will be able to write valid mathematical arguments.
3. Students will exhibit familiarity and ease of use with sequences and series.
4. Students will recognize, have knowledge of, be able to combine and evaluate mathematical statements involving quantifiers.
5. Students will exhibit familiarity and ease of use with the definitions of continuity and differentiability.

Assessment Method: Embedded questions throughout homework, quizzes and exams.

Illustration 1 *What is the contrapositive of this statement:*

$$\exists\varepsilon,\ a-\varepsilon \geq b \iff a > b.$$

Show that the statement is false.

It is expected is the student will be able to read and interpret the symbols (1) and have sufficient command over them to form the contrapostive (4):

$$\forall\varepsilon,\ a-\varepsilon < b \iff a \leq b.$$

Noting that ε is not restricted, a choice of $\varepsilon = a - b$ makes the left side of the statement false, therefore the implication $\forall\varepsilon,\ a-\varepsilon < b \implies a \leq b$ is false, and thus the full statement is false (2).

Illustration 2 *Show that the function $f(x) = 1/x$ is continuous, but not uniformly continuous, on $0 < x \leq 1$.*

It is expected is that student will be able to apply the $\varepsilon - \delta$ definition to this problem, find that that $0 < \delta < \varepsilon x^2/(1-\varepsilon x)$ and that $0 = \inf\{\varepsilon x^2/(1-\varepsilon x) : 0 < x \leq 1\}$ so that there is no uniform δ (5)(1)(4)(2).

Illustration 3 *Show that a bounded monotone increasing sequence converges to it's supremum.*

It is expected that the student will be able to read and decode this statement into (1)(3)

$$\exists M > 0 \text{ s.t. } \forall n,\ a_n < M \text{ and } a_n < a_{n+1} \implies \lim_{n\to\infty} a_n = \sup_n a_n.$$

From this, further decode the conclusion into (1)(3)

$$\forall \varepsilon > 0,\ \exists N \text{ s.t. } |a_n - \sup_n a_n| < \varepsilon\ \forall n > N.$$

A consequence of the sequence being bounded is that the supremum is finite, and using the definition of finite supremum (1)(3),

$$\forall \varepsilon > 0,\ \exists N \text{ s.t. } \sup_n a_n - \varepsilon < a_N \text{ and } a_n \leq \sup_n a_n,\ \forall n.$$

The result follows using monotonicity in that statement (2)(3):

$$\forall \varepsilon > 0,\ \exists N \text{ s.t. } \sup_n a_n - \varepsilon < a_N < a_n \leq \sup_n a_n,\ \forall n > N.$$

MATH 364A ORDINARY DIFFERENTIAL EQUATIONS I

Goals:

1. Students will learn, understand, and use the fundamental concepts of differential equations.

2. Students will solve problems involving differential equations.

3. Students will read, understand, and use mathematical reasoning and symbolism, and employ them to communicate their ideas in a logical organized manner.

4. Students will examine applications of differential equations to science and engineering and learn to model problems in these areas using differential equations.

5. Students will study the theory of differential equations, especially the use of ideas from calculus and from linear algebra in its development.

Assessment Method: Embedded questions throughout homework, quizzes, and exams. Discourse in class and office hours.

Illustration #1: In a nefarious scheme to render Superman helpless, Lex Luthor immerses him in a tank containing 12 lb of Kryptonite dissolved in 100 gal of water. Fortunately, Lois Lane is able to puncture the tank so that the mixture flows out of the tank at a rate of 5 gal/min. Unfortunately, Luthor is pouring water containing 2 lb of Kryptonite per gallon into the tank at a rate of 4 gal/min. How much Kryptonite is in the tank one hour later?

The student starts by defining $A(t)$ to be the amount of Kryptonite in lbs in the tank at time t minutes after Superman is immersed.[3] The student sees that $\frac{dA}{dt}$ is the rate at which this amount changes in lb/min and must equal the rate at which Kryptonite enters the tank minus the rate at which Kryptonite leaves the tank. The rate at which Kryptonite enters the tank is $(2\,lb/gal)\,(4\,gal/min) = 8\,lb/min$. To find the rate at which

Kryptonite leaves the tank, the student calculates the number of gallons in the tank at time t to be
$100\,gal \;+\; \left(4\,\dfrac{gal}{min} \;-\; 5\,\dfrac{gal}{min}\right)(t\,min) \;=\; 100 - t\,gal$, and concludes that the rate is
$\left(\dfrac{A(t)\,lb}{100 - t\,gal}\right)\left(5\;\dfrac{gal}{min}\right) \;=\; \dfrac{5A}{100{-}t}\;\dfrac{lb}{min}$ when $0 \le t \le 100$.[4] Thus the student arrives at the differential equation $\dfrac{dA}{dt} = 8 - \dfrac{5A}{100 - t}$. Recognizing this as a linear first order equation, the student rewrites it in the form $A' + \left(\frac{5}{100-t}\right) A \;=\; 8$ and computes the integrating factor
$e^{\int \frac{5}{100-t}\,dt} \;=\; e^{-5\,ln\,(100-t)} \;=\; (100-t)^{-5}$.

Upon multiplying both sides of the differential equation by the integrating factor, the student obtains

$$\left[(100-t)^{-5}A\right]' \;=\; (100-t)^{-5}\left[A' + \frac{5}{100-t}\,A\right] \;=\; (100-t)^{-5}\,8$$

after which both sides are integrated to get
$(100-t)^{-5} A \;=\; 2\,(100-t)^{-4} \;+\; c$. The student simplifies this to get
$A(t) \;=\; 2(100-t) \;+\; c(100-t)^5$.[1,2]

Since there were originally 12 lb of Kryptonite in the tank, the student plugs into the above equation to get $12\,lb \;=\; A(0) \;=\; 2(100) \;+\; c\,(100)^5$ and simplifies to get $c = \dfrac{-188}{(100)^5}$.

Noting that one hour is 60 minutes, the student concludes that the amount of Kryptonite in the tank one hour later is
$A(60) \;=\; 2(100-60) \;-\; \dfrac{188}{(100)^5}\,(100-60)^5 \;=\; 78.1\,lb$.[2,4]

Illustration #2: Suppose $u(t)$ and $v(t)$ are twice differentiable functions whose Wronskian is $W(t) \;=\; (t^2 + 4t + 4)\,e^{7t}$. Are $u(t)$ and $v(t)$ linearly independent? Do there exist continuous functions $p(t)$ and $q(t)$ such that $u(t)$ and $v(t)$ form a fundamental set of solutions of the differential equation $y'' + p(t)\,y' + q(t)\,y \;=\; 0$ in the interval $-\infty < t < \infty$?

Referring to the theorem that states that if the Wronskian of u and v exists and is nonzero at some point in an interval I, then u and v are linearly independent on I, the student ascertains that u and v are linearly independent on $-\infty < t < \infty$.[3,5] Referring to Abel's Theorem, the student recognizes that, when p and q are continuous on an interval I, the Wronskian of two solutions of the differential equation $y'' + p(t)\,y' + q(t)y = 0$ must be either always equal to zero or never equal to zero on I.[1] In this case, I is $-\infty < t < \infty$ and the Wronskian $W(t) = (t^2 + 4t + 4)\,e^{7t} = (t+2)^2\,e^{7t}$ equals zero at $t = -2$ and does not equal zero elsewhere, so the student concludes that u and v cannot both be solutions of the differential equation and hence cannot form a fundamental set of solutions of the differential equation.[3,5]

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Goals:

1) Students will formulate real world situations in meaningful mathematical forms, including frequency interpretation of probability, diagrams or equations.
2) Students will recognize axioms of probability theory and be able to apply them to real-world situations.
3) Students will be able to use counting techniques to compute elementary probability estimates.
4) Students will be able to evaluate probabilities involving continuous density functions and recognize their uses.
5) Students will execute statistical manipulation and computation with random variables and statistical functions in order to solve a posed problem.
6) Students will interpret statistical results about real-world situations.

Assessment Method: Embedded questions throughout homework, quizzes and exams.

Illustration #1: Baye's theorem is very good when applied to diagnosing medical conditions. When the disease being checked for is rather rare, the number of incorrect diagnoses can be surprisingly high.

Consider "screening" for cervical cancer. Let A be the event of a woman having the disease and B, the event of a positive biopsy. So, B occurs when the diagnostic procedure indicates that she does have cervical cancer. Assume that the probability of a woman having the disease, P(A), is 0.0001, that is 1 in 10,000. Also, assume that given that the person has the disease, the probability of a positive biopsy is 0.90. So, the test correctly identifies 90% of all women who do have the disease. Also, assume that if the person does not have the disease, the test incorrectly says that person does have the disease 1 out of every 1000 patients.
1) Find the probability that a woman has the disease given that the biopsy says she does.

The student has to recognize this problem as a Baye's Theorem problem. In order to solve such a problem, the student has to be organized and write down everything that was given in the problem and then write down in symbols what is being asked to solve.

Using A and B as described in the problem, the student should be able to see that the question can be written as: find $P(A|B)$. Also, from the problem, the student can see that

P(A)=0.0001, (and therefore $P(A^c)$=1-P(A)=1-0.0001=0.9999).
Also, the student should see that P(B|A)=0.90.
Finally, the student should see that $P(B|A^c)$=0.001.

Then, using Baye's Theorem, the student should realize the following is equal to P(A|B):

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$

$$= \frac{(0.9)(0.0001)}{(0.9)(0.0001) + (0.001)(0.9999)}$$

= 0.08

So, only 8% of the women identified as having the disease actually do have the disease. This is an alarmingly low accuracy rate.

2) Now, assume we have a more common disease. Assume that the probability of a woman having the disease, P(A), is 0.01, that is 1 in 100. Now, find the probability that a woman has the disease given that the biopsy says she does. That is, find P(A|B).

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$

$$= \frac{(0.9)(0.01)}{(0.9)(0.01) + (0.001)(0.99)}$$

= 0.90

So, 90% of the women identified as having the disease actually do have the disease. This is a much higher accuracy rate.

These questions combined show the prevalence of a disease has a major influence on the accuracy of the tests.

Illustration #2: Any person who has an IQ in the upper 2% of the general population is eligible to join the international society devoted to intellectual pursuits called Mensa.

1) Assuming IQ's are normally distributed with mean of 100 (i.e., μ=100) and a standard deviation of 16 (i.e., σ=16) what is the lowest IQ that will qualify a person for membership?

Let the random variable Y be the person's IQ. Let y_L be the lowest IQ that qualifies someone to be a member of Mensa. So, the student should see that we are looking for $P(Y \geq y_L)=0.02$.

First, the student should realize that standard normal charts are based on cumulative probabilities. So, the student needs to look up
$P(Y < y_L) = 1 - 0.02 = 0.98$.

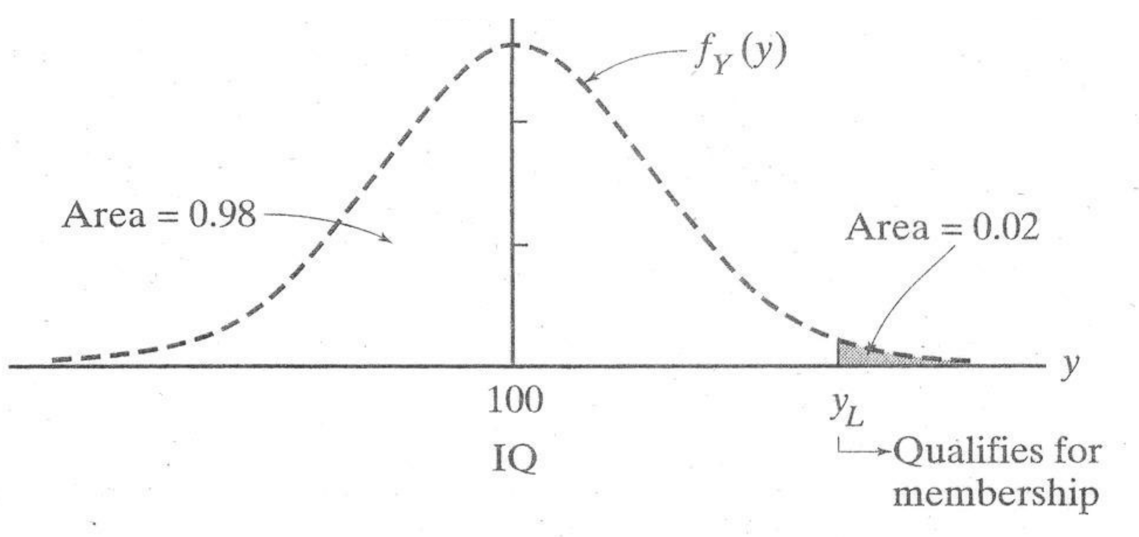
Next, the student needs to know how to standardizing the values to read off a standard normal chart.

In order to standardize, the student has to realize that $Z = \dfrac{Y - \mu}{\sigma}$.

Using this, the student should get

$$P(Y < y_L) = P\left(\frac{Y-100}{16} < \frac{Y_L-100}{16}\right) = P\left(Z < \frac{Y_L-100}{16}\right) = 0.98$$

So, from a standard normal table, the student can see by looking inside the chart, the Z value that corresponds to 0.98 is Z=2.05.
So, setting equal 2.05= $\dfrac{Y_L-100}{16}$, we get y_L=100+16(2.05) = 133



So, a person needs to get an IQ score of at least 133 to be able to join Mensa.

Illustration #3: As the lawyer for a client accused of murder, you are looking for ways to establish “reasonable doubt” in the minds of jurors. The prosecutor has testimony from a forensics expert who claims that a blood sample taken from the scene matches the DNA of your client. One out of 1000 times such tests are in error.

Assume your client is actually guilty. If six other laboratories in the country are capable of doing this kind of DNA analysis and you hire them all, what are the chances that at least one will make a mistake and conclude that your client is innocent?

Each of the six laboratories constitutes an independent trial, where the probability of making a mistake, p, is 0.001 (i.e., 1 in 1000). Let X=the number of labs that make a mistake (and call your client innocent).

The student should be able to recognize this as a discrete distribution, where X is a binomial with the sample size, n, equal to 6 and p=0.001. So, P(X=k) = $\binom{n}{k}(0.0001)^k(0.999)^{n-k}$

P(at least 1 lab says the client is innocent)=P(X ≥ 1)

The student should know that the sum of all the probabilities in a discrete distribution always equal 1. So, in this case,
P(X=0)+P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)+P(X=6)=1

Consequently,
P(X ≥ 1) = P(X=1)+P(X=2)+P(X=3)+P(X=4)+P(X=5)+P(X=6)

Or, equivalently and less computationally intensive,
P(X ≥ 1) = 1 – P(X < 1)
= 1- P(X=0)
$= 1 - \binom{6}{0}(0.0001)^0(0.999)^6$ = 0.006.

So, there is only a 0.6% chance of the lawyer’s strategy working.

Math 381, Mathematical Statistics

Goals:

1. Students will utilize the probability theory as well as distribution theory as a foundation for the statistical inference.
2. Students will recognize proper statistical inference procedure(s) for a given problem.
3. Students will understand and recognize the underlying assumptions for statistical inference methods.
4. Students will use cutting-edge computer technology for data analysis through statistical inference methods.
5. Students will be able to implement the language of statistical methodology for appropriate communication.
6. Students will be able to demonstrate the mathematical constructions of the point and interval estimation (confidence interval) as well as hypothesis testing.
7. Students will be able to interpret the results of statistical analyses in the language of the given problem.

Assessment Method: Embedded questions throughout homework, projects, and exams.

Illustration #1: Chronic anterior compartment syndrome is a condition characterized by exercise-induced pain in the lower leg. Swelling and impaired nerve and muscle function also accompany the pain, which is relieved by rest. Susan Beckham and colleagues [S.J. Beckham, W.A. Grana, P. Buckley, J.E. Breasile, and P.L. Claypool, "A Comparison of Anterior Compartment Pressures in Competitive Runners and Cyclists," *American Journal of Sports Medicine* 21 (1) (1993)] conducted an experiment involving ten healthy runners and ten healthy cyclists to determine if pressure measurements within the anterior muscle compartment differ between runners and cyclists. The data—compartment pressure, in millimeters of mercury—are summarized in the following table.

	Runners		*Cyclists*	
Condition	*Mean*	*s*	*Mean*	*s*
Rest	14.5	3.92	11.1	3.98
80% maximal O_2 consumption	12.2	3.49	11.5	4.95

a. Construct a 95% confidence interval for the difference in mean compartment pressures between runners and cyclists under the resting condition.

The student recognizes that the problem is requesting information related to comparison of two means **(2)**.

Let $\mu_1 =$ mean compartment pressure for all runners under resting condition.
$\mu_2 =$ mean compartment pressure for all cyclists under resting condition.

The small-sample 95% confidence interval for $\mu_1 - \mu_2$ is

$$(\overline{x}_1 - \overline{x}_2) \pm t_{\alpha/2,\, n_1+n_2-2} \;\; S_p \sqrt{\tfrac{1}{n_1} + \tfrac{1}{n_2}}$$ **(2), (5)**

The student mentions the underlying assumptions of normality and equal variances and explains the notations involved in this process. **(3), (1)**

Numerical calculations can be performed by computer statistical packages or advanced graphing calculators. **(4)** Student reports the results of calculations

$$n_1 + n_2 - 2 = 18 \;\; , \quad \geq \tfrac{\alpha}{2} = 0.025 \;\; , \;\; t_{0.025,\, 18} = 2.101$$

$$S_p^2 = \tfrac{9(3.92)^2 + 9(3.98)^2}{18} = 15.6034$$

$$(14.5 - 11.1) \pm (2.101) \sqrt{15.6034\left(\tfrac{1}{10} + \tfrac{1}{10}\right)} \;\Rightarrow (\,-0.3\,,\,7.1)$$

b. Construct a 90% confidence interval for the difference in mean compartment pressures between runners and cyclists who exercise at 80% of maximal oxygen consumption.

The student recognizes that in this case the comparisons between means is requested for runners and cyclists who exercise at 80% of maximal oxygen consumption. **(2)**

The student also recognizes that similar assumptions and notations can be implemented in this case. **(3), (1)**

Similarly, the student reports the results of calculations
(4), (5)

$$n_1 + n_2 - 2 = 18 \;\; , \;\; \tfrac{\alpha}{2} = 0.05 \;\; , \;\; t_{0.05,\, 18} = 1.734$$

$$s_p^2 = \tfrac{9(3.49)^2 + 9(4.95)^2}{18} = 18.3413$$

$$(12.2 - 11.5) \pm (1.734) \sqrt{18.3413\left(\tfrac{1}{10} + \tfrac{1}{10}\right)} \;\Rightarrow (\,-2.62\,,\,4.02)$$

c. Consider the intervals constructed in (a) and (b). How would you interpret the results you obtained?

The student notices that both intervals above contain 0, as a result it cannot be concluded that a difference exists in mean compartment pressure between runners and cyclists in either condition. **(7)**

d. Is there sufficient evidence to justify claiming that a difference exists in the mean compartment pressure between runners and cyclists who are resting? Use $\alpha = .05$. What can be said about the associated p-value?

In this case, the student recognizes that the language of the problem is hypothesis testing. **(2)**

The student formulates the problem and performs necessary computations. **(1), (3), (4), (5)**

$$H_0: \ \mu_1 - \mu_2 = 0$$
$$H_a: \ \mu_1 - \mu_2 \neq 0$$

The test statistic is

$$t = \frac{(\overline{x}_1 - \overline{x}_2) - 0}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{14.5 - 11.1}{\sqrt{15.6034\left(\frac{1}{10} + \frac{1}{10}\right)}} = 1.92$$

The rejection region is $|t| > t_{.025,18} = 2.101$.

The value of $t = 1.92$ does not belong to the rejection region. The student reports that H_0 is not rejected, i.e., there is insufficient evidence to indicate a difference in the means of compartment pressure for all runners and cyclists at resting condition. **(7)**

The student can report the p-value as $0.05 < p\text{-value} < 0.10$, that is consistent with the conclusion given above. **(5)**

> **e**. Does sufficient evidence exist to permit us to identify a difference in the mean compartment pressure between runners and cyclists at 80% maximal O_2 consumption? Use $\alpha = .05$. What can be said about the associated p-value?

The student recognizes that here the testing is between the means of compartment pressure of runners and cyclists at 80% maximal O_2 consumption. **(2)**

The student also recognizes that similar assumptions and notations can be implemented in this case. **(3), (1)** Similarly, the student formulates the problem in the language of hypothesis testing and performs necessary computations. **(1), (3), (4), (5)**

The test statistic is

$$t = \frac{12.2 - 11.5}{\sqrt{18.3413\left(\frac{1}{10} + \frac{1}{10}\right)}} = 0.365$$

The rejection region is $|t| > t_{0.02,18} = 2.101$. Since the value of $t = 0.365$ does not belong to the rejection region, the student concludes that there is insufficient evidence to indicate a difference in the mean compartment pressure between runners and cyclists at 80% maximal O_2 consumption. **(7)**

The associated p-value is reported as

p-value > 0.20 . **(5)**

Illustration #2:

Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the normal distribution with $E(Y_1) = \mu$ and $V(Y_1) = \sigma^2$. We wish to estimate the value of σ^2.

a. Define

$$V = \frac{(n-1)S^2}{\sigma^2} ,$$

where $S^2 = \frac{1}{(n-1)} \sum_{i=1}^{n} \left(Y_1 - \overline{Y}\right)^2$. State the distribution of V.

The student recognizes that V is distributed as chi-square with $n-1$ degrees of freedom, i.e., $V \sim X^2(n-1)$. **(1)**

b. The random variable V defined in **a.** is pivotal quantity. Make use of V to derive a $(1-\alpha)$ 100% one-sided upper confidence interval for σ^2 (i.e., a CI of the form $0 < \sigma^2 \le \widehat{\sigma}_U^2$). Also derive a $(1-\alpha)$ 100% one-sided upper confidence interval for σ.

By using the information in part **a.**, the student formulates the problem. **(1), (2), (6)**

$$\begin{aligned} 1-\alpha &= P\left(X^2_{1-\alpha\,,n-1} \le V\right) = P\left(X^2_{1-x,n-1} \le \frac{(n-1)S^2}{\sigma^2}\right) \\ &= P\left(\sigma^2 \le \frac{(n-1)S^2}{x^2_{1-\alpha,n-1}}\right) \end{aligned}$$

Thus, $(1-\alpha)$ 100% CI for σ^2 is $0 < \sigma^2 < \frac{(n-1)s^2}{x^2_{1-\alpha,\,n-1}}$, where s^2 is the value of S^2 calculated from the sample. **(5)**

By taking the square root, the student obtains a $(1-\alpha)$ 100% CI for σ as $0 < \sigma < \sqrt{\frac{(n-1)\,s^2}{X^2_{1-\alpha,\,n-1}}}$. **(5)**

c. A company wishes to experimentally establish the amount of variability to expect from

a precision measuring device that they manufacture. To accomplish this, they select at random 30 identical such meters and use them to make measurements of the same item. Thus, except for the inherent variability in the meters, all measurements should be the same. The data so obtained are assumed to be normally distributed with mean μ and variance σ^2. The data may be summarized as follows: $n = 30$, $\overline{y} = 5.136$ units, and $s^2 = 0.001345$ squared units. Compute a 90% one-sided upper confidence interval for σ.

From part **b.**, the student calculates a 90% CI for σ as

$$0 < \sigma < \sqrt{\frac{(30-1)(0.001345)}{19.7677}} \Rightarrow 0 < \sigma < 0.0444 \qquad \textbf{(4)}$$

The student reports that the interval constructed in this manner encloses the amount of variability expected from a precision measuring device that the company manufactures 90% of the time in repeated sampling. Hence, we are fairly certain that this particular interval encloses σ . **(7)**

Self Review **Math 410** **April 2006**

Goals

1) Students will examine the development of modern mathematics with an emphasis on understanding the connections between problems, ideas, and theories. This can include reading and discussing original material.
2) Students will analyze and write with clarity, coherence, and concision. They will consider problems in their historical context, formulate hypotheses that account for theoretical approaches to these problems, and support their claims with arguments tied to original or secondary sources.

Assessment Method 1. Written responses to specific questions. Can be in the form of an essay or problem or a mixture.

Illustrations.

1) Given a regular polyhedron, the *circumsphere* is the sphere that passes through the vertices (the smallest sphere that contains the polyhedron) and the *insphere* is the sphere that passes through the face-centers (the largest sphere that the polyhedron contains).

   In *The Harmony of the World*, Kepler describes a model for the solar system that's based on the five platonic solids. Briefly discuss how this model works. (The circumsphere and insphere terminology might help.) To what sort of paradigm of explanation does this appeal? How does the *spirit* of this approach fit into the way in which science was developing at that time?

   Find the ratio of the radii of the circumsphere and insphere for the cube, regular octahedron, and regular tetrahedron.
2) Use Descartes's method of normals to find the slope of the line tangent to the curve given by $y = x^3$ at an arbitrary point (x, y). Use Fermat's method of adequation to find the same result. (You needn't follow their procedure exactly. You can use more modern notation and ideas to convey the ideas.) What are the advantages and disadvantages of each compared to the other? What conceptual innovations make their methods possible?
3) By means of an illustrative problem or two, discuss the similarities and differences between the notions of indivisibles and infinitesimals. (One approach would be to solve a problem using each idea.) Treat the application of these concepts to both the problem of finding tangents to curves and to the classic problems of finding the area bounded by curves. What advantage does one have over the other? What are their relative disadvantages?
4) Use Newton's *general* binomial theorem to compute—in the manner of the fluxional calculus—the first fluxion of the fluent

   $$x^{p/q}$$

   where p and q are integers. Carefully discuss Newton's conception and use of fluent quantities and their fluxions. What are the crucial or suspicious features of this derivation?

5) Use a *specific* and *relatively simple* example to present Cramer's paradox for algebraic curves. Carefully discuss what's paradoxical here. In terms of your example, illustrate how to resolve the puzzle. In a theoretical sense, what makes the paradox possible? (Could the problem have occurred in classical Greece, say?)
6) Without giving technical details, how does Newton go about deriving and "proving" the generalized binomial theorem? By the standards of the time, to what extent can the two activities—derivation and proof—be distinguished? How does Newton's work here compare and contrast with mathematical work today?

Assessment Method 2. In-class presentations on a reading or problem.

Illustrations of what students will consider and treat.

1) What's the problem at hand? How does the author approach it? What ideas and techniques are in use—both explicitly and implicitly? What's the intellectual, technological, and social context?
2) What concepts does the reading rely upon? What's the author's *point of view*? What assumptions does the author make? What philosophical issues and historical questions does the reading suggest? What does the reading tell us about how math was done at the time?
3) What are the crucial points in the development of an idea or theory?
4) What new ideas are involved? How do they arise from and connect with existing notions and theories—particularly ones that we've seen in previous readings? What sort of conceptual change is taking place?
5) What role does language and notation play in the attempt to explain things?
6) In what ways did the ideas and developments in the reading influence subsequent mathematics?

Assessment Method 3. Independent course project with substantive mathematical and historical content on a topic that is somewhat beyond the class material.

Illustrations.

1) In what ways did the development of the 'function' concept influence coordinate geometry? Compare algebraic and graphical representations of functions.

   Source: C. Boyer, *History of analytic geometry*
2) Investigate the interplay between mathematics and the making of maps. What problems did the latter pose for the former? How did this interaction influence the development of mathematical theory?
3) Examine the development of some of the statements in plane geometry that are *equivalent* to Euclid's parallel postulate. (See below.) How was a specific statement found to be equivalent to the parallel postulate? What impact on geometric thought did these discoveries have? Show how the parallel postulate comes into play.

- There are non-congruent similar triangles.
- Three points determine a circle.
- The angle sum of any triangle is π radians.
- There are such things as rectangles.

4) Examine paradoxes of indivisibles—arguments based on indivisibles that “show” that two figures having different areas have the same area. In response, how was the method of indivisibles defended? What do these cases tell us about mathematical thought and proof? In what ways did these controversies influence the development of math?
5) An indivisible can be thought of as a “sum-of-limit” approach to integration. Describe this way of looking at the issue and compare it to Riemann’s “limit-of-sums” method.
6) How did Zeno’s paradoxes influence mathematical and philosophical thought in the Renaissance or the Enlightenment periods. In particular, to what extent was calculus a response to the paradoxes?
7) How did the evolution of calculus depend on and influence ideas of infinity? Was a particular notion of “the infinite” required for calculus to develop beyond a certain point?
8) Archimedes carefully distinguished methods of discovery and proof. Can one make a similar distinction regarding infinitesimal and “limit” approaches to calculus? Might the method of infinitesimals have been a tool of discovery while “rigorous” argument required limit-taking or exhaustion? Exhibit this sort of distinction with an example where one must know the answer in advance in order to make a limit argument.
9) The Hughes-Hallet, Gleason text *Calculus* (the so-called “Harvard calculus”) introduces integration as the inverse of differentiation. In many (most?) modern texts this relationship appears as a result of defining the integral as an area or Riemann sum. Study the history of presentations of integration and the fundamental theorem of calculus. Is the Harvard approach new? What’s your opinion concerning the effectiveness of the various styles?
10) Modern math often appears as the work of an elite class of white European men. What social conditions provided for and promoted this state of affairs? What influence did this bias have on education in general and math education in particular? To what extent could developments in non-European cultures have influenced thinking in Europe? Speculate on how the history of math could have been quite different had this neglect not taken place?
11) In the 17th century, scientific societies emerged for the first time—principally, in France, Italy, and England. How did they function and what did they promote? In what ways and to what extent did they foster new thinking and, contrarily, in what ways and to what extent did they restrict the development of new ideas and theories?
12) Prepare a detailed plan for a high-school math course (or lesson) that involves (modern) historical issues? What pedagogical role do the historical features play? What activities will the students engage in? How does this approach promote understanding and *deepen* the learning experience for students? What do you want students to get from the course (or lesson)?

    Source: F. Swetz, ed. *Learn from the masters!*

MATH 444: Introduction to Abstract Algebra

Course description

1 Target audience

MATH 444 is required for all students studying for a Bachelor of Science in Mathematics (the "general option") or for a Bachelor of Science in Mathematics: Option in Mathematics Education. These students, usually seniors, have mostly already had an introduction to techniques of proof in MATH 233 and a previous course in number theory or linear algebra, MATH 341 or MATH 347.

MATH 444 is also sometimes recommended by the Graduate Advisor for students in our Masters program who may not be prepared for MATH 540A.

2 Goals

- Familiarity with specific topics.

  See the list of topics below.

- Practice in studying abstract systems defined by formal axioms.

  Students also receive such practice in other courses, notably with the definitions of vector spaces in MATH 247 and MATH 347 and of metric spaces in MATH 361B. However, this idea dominates MATH 444 from beginning to end. Every MATH 444 student encounters formal definitions of groups and rings, and many courses also cover such variations at semigroups, monoids, fields, domains, algebras, principal ideal domains, and euclidean domains. The repetition of the pattern teaches students that we are not simply introducing them to specific structures. Rather, we are cultivating the skill of using formal axioms to capture the essential qualities of an object and to find common principles among disparate examples. We then work from these formal axioms to prove theorems that are then valid for all the examples.

- Training in writing proofs.

  Many mathematics courses at CSULB include some proofs, but the ones that are most focused on proofs are MATH 233, MATH 361AB, and MATH 444. MATH 233 is a lower-division transition course designed to prepare students for later work. MATH 361 focuses on the "$\delta - \epsilon$" style of proofs used in analysis. Students majoring Math Education are not required to take MATH 361, so for these students, MATH 444 is the most sophisticated training they will receive in writing proofs.

- Preparation for graduate-level algebra.

  Every Masters and Ph.D. program in mathematics includes a core algebra course as a staple component of first year study. (At CSULB, this is the MATH 540AB sequence; at UCLA, it is MATH 210ABC.) Although these courses generally develop group theory from scratch, they move at such a rapid pace that in practice, students need to have had some prior exposure to both the material and the methods of algebra.

- A formal introduction to symmetry.

  The symmetry of a formal system (for example, a geometric object such as a square) is a concept that many students grasp intuitively, and many of our Math Education majors will explore the idea with their high school students. In MATH 444 we develop formal frameworks in which such ideas can be studied.

3 Topics

The following topics are always covered in MATH 444:

- groups
  - subgroups
  - common examples
    - symmetric groups
    - dihedral groups
    - the Klein 4-group
    - the finite quaternion group
    - the unit group of the integers mod n
  - cyclic groups
  - cosets
  - Lagrange's Theorem
  - normal subgroups
  - quotient groups
  - homomorphisms and isomorphisms of groups
  - kernel and image
  - the Fundamental Homomorphism Theorem
- rings
  - the integers and the integers mod n
  - polynomial rings
  - integral domains
  - ideals
  - quotient rings
  - homomorphisms of rings

Here are some topics that may be covered in MATH 444:

- matrix groups
- alternating groups
- direct products
- simple groups
- Cayley's Theorem
- Cauchy's Theorem
- conjugacy
- semigroups
- monoids
- fields
- algebras
- euclidean domains
- prime and maximal ideals
- characteristic
- finite fields
- principal ideal domains

4 Assessment

Students in MATH 444 always complete homework assignments and exams. Other methods of assessment can include quizzes, independent projects, and classroom presentations. In all methods, students both calculate examples and prove general theorems.

We present two illustrations of problems that were assigned as homework in Spring 2005, with comments on their intended purpose.

1. Show Cayley's Theorem in action by finding an isomorphic copy of the finite quaternion group Q_8 in the symmetric group S_8:
   (a) Write the elements of Q_8 in the form $\{e, i, i^2, i^3, j, ij, i^2j, i^3j\}$.

(b) Figure out which two permutations in S_8 correspond to i and j, as we did in class. (You don't have to do this for every element of Q_8, just i and j, since every other element can be written in terms of i and j. In other words, i and j *generate* Q_8.)

(c) Now that you know $\varphi(i)$ and $\varphi(j)$, figure out the permutations corresponding to the other elements of Q_8 by using the homomorphism law $\varphi(g_1 g_2) = \varphi(g_1)\varphi(g_2)$.

(d) We know that in Q_8, i and j satisfy the relations $i^4 = e, j^4 = e$, and $ji = (i^2)ij$. Check that these same relations are satisfied for the $\varphi(i)$, and $\varphi(j)$ you found in S_8.

Comments:

A common quandary in theoretical courses is how to convey the workings of the proof of a major theorem to the students. We can force them to memorize the proof and regurgitate it, but they may still not understand it. We can have them solve problems using the theorem, but many theorems can be applied with no knowledge at all of why they are true. We can ask them to prove similar theorems, but the ideas may be too sophisticated to expect students to come up with them on their own.

This problem, which can be solved by fairly easy calculation, is an attempt to get students to understand the proof of Cayley's Theorem, which involves representing a group as a set of transformations on an external set (in this case, the group itself). Students must work through the proof in the context of a specific example, the finite quaternion group.

On a deeper level, we are trying to inculcate the concept that every group can be considered as a group of transformations.

(a) This part gives students practice calculating in the finite quaternion group, an important example in abstract algebra. It also reminds them of the process of trying to write all elements in a group in terms of as few generators as possible. Finally, it establishes an order for the elements of the group, which is necessary to fix a particular representation in the symmetric group. Since we are giving them a particular order, all students will use the same order, so they can check their answers to the calculations in later parts with other students.

(b) To solve this part, students must first do a significant amount of calculation, multiplying i and j by every other element of the group and then simplifying using the generating formulas. They must then translate their answers into permutations in the symmetric group.

The amount of calculation involved is enough to convince the students that doing it for all eight elements of Q_8 would be truly cumbersome. This reinforces the power of using generators to control all elements of a group.

(c) This reminds students of the homomorphism law and forces them to use it extensively. It also gives them lots of practice in multiplying permutations in a nontrivial symmetric group. (Most examples in MATH 444 do not go beyond S_3 or S_4.) It illustrates the point that because of the homomorphism law, a homomorphism is determined solely by its action on the generators of a group. Finally, it gives them a concrete set of permutaions in S_8 that correspond to the image of Q_8.

(d) This gives students further practice in calculating in the symmetric group. They are checking that their subset of S_8 does indeed behave identically to Q_8. This also allows them to check all their previous work, so that by the time they are finished, they will be almost certain that everything is right.

More importantly, this teaches the idea that groups are determined not just by their generators, but by a set of relations, and it is usually preferable to make the relations as brief as possible. In this case, Q_8 was originally defined using the relation $ji = -k$; this shows students that the same relation can be written in terms of i and j only. In order to check that a homorphism is well-defined, it is necessary to check that the relations on the generators of the domain are still valid in the image.

2. Suppose a, b are two elements in a ring such that $ba = 1$ but $ab \neq 1$.

    (a) Prove that the element $v = 1 - ab$ is *idempotent*, i.e. that $v^2 = v$. Prove that v is not equal to either of the trivial idempotents, 1 or 0.

    (b) Prove that for all $n = 0, 1, 2, \ldots$, the element $c_n = a^n v + b$ is a left inverse for a.

    (c) Prove that if $c_n = c_m$, then $n = m$. (Hint: If $n > m$, then left multiply the equation $c_n = c_m$ by b^n.) Conclude that if an element is left invertible but not right invertible, then it has infinitely many left inverses!

    (d) For the ring of infinite matrices we studied in class, calculate what each c_n is.

Comments:

This problem walks the students through a relatively lengthy proof. Most of the mechanics can be done by straightforward calculation, but the logic is subtle at times. It gives them concrete practice working in a system in which one of the long cherished principles of elementary arithmetic, commutativity, does not hold. For the first three parts of the problem, they must work in an abstract system, and in the final part, they calculate a concrete example of what they have proved.

(a) The students first learn a new concept, *idempotence*, which is introduced in the context of the problem rather than in a formal definition. They must immediately use the definition in a proof. The actual proof is a quite simple calculation following immediately from the definition.

    A common reaction from students upon seeing the equation $v^2 = v$ is to factor $v(v-1) = 0$ and conclude that $v = 0$ or $v = 1$. This problem forces students to challenge that assumption and recognize the existence of nonzero elements that multiply to zero. In fact, they must prove that $v \neq 0$ and $v \neq 1$. The former is trivial from the assumption that $ab \neq 1$, but the latter requires a slightly longer chain of logic.

(b) The phrase "for all $n = 0, 1, 2, \ldots$" makes this look like an induction problem, and many students will immediately try to apply some form of induction. In fact, it is a simple calculation following from the definition of left inverse. One of the lessons of our algebra courses is that many proofs can be derived simply by careful invocation of the definitions of the terms in the hypothesis and conclusion.

(c) This proof would be quite tricky without the hint. Even with the hint, students must argue the contrapositive, recognize the symmetry between $n > m$ and $m > n$, do a nontrivial calculation, and finally realize that the result $v = 0$ contradicts something that they had proved in an earlier part of the problem.

Students must then recognize the import of what they have just proved, namely that the elements c_n are all different and that therefore the element a has infinitely many left inverses. Again, this violates a cherished assumption, uniqueness of inverses, that has held true through the linear algebra they saw in MATH 247 and even as recently as the group theory that they saw earlier in MATH 444.

(d) Students had at this point already seen the following example in class:

$$a = \begin{bmatrix} 0 & 0 & 0 & & \\ 1 & 0 & 0 & \cdots & \\ 0 & 1 & 0 & & \\ & \vdots & & \ddots & \end{bmatrix}, b = \begin{bmatrix} 0 & 1 & 0 & & \\ 0 & 0 & 1 & \cdots & \\ 0 & 0 & 0 & & \\ & \vdots & & \ddots & \end{bmatrix}$$

They must now calculate

$$v = \begin{bmatrix} 1 & 0 & 0 & & \\ 0 & 0 & 0 & \cdots & \\ 0 & 0 & 0 & & \\ & \vdots & & \ddots & \end{bmatrix}, c_0 = \begin{bmatrix} 1 & 1 & 0 & & \\ 0 & 0 & 1 & \cdots & \\ 0 & 0 & 0 & & \\ & \vdots & & \ddots & \end{bmatrix}, c_1 = \begin{bmatrix} 0 & 1 & 0 & & \\ 1 & 0 & 1 & \cdots & \\ 0 & 0 & 0 & & \\ & \vdots & & \ddots & \end{bmatrix}, \ldots.$$

These relatively simple matrices give a tangible representation of the elements calculated in the first three parts of the problem. They also make it somewhat intuitively clear that an element may have infinitely many distinct inverses.

Students with better training from linear algebra will understand the correspondence between matrices and linear transformations on a vector space. Under this correspondence, all the equations from the first three parts make intuitive sense. The initial elements a and b are right and left shift operators, which are inverses of each other on one side, but not on the other. v is projection to the first coordinate axis, and a projection is always idempotent even through it may not be the identity or the zero map. The right shift has many left inverses, since it does not matter what happens to the first coordinate axis after the right shift is applied and before the left shift is applied.

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Goals:

- ¬ Students will be familiar with the physical significance of the basic partial differential equations of physics: the Laplace equation, the wave equation, and the heat equation.
- ❷ Students will be able to classify second-order partial differential equations.
- ❸ They will be able to solve transport equations and understand their connection to wave propagation.
- ❹ They will understand the significance of the auxiliary conditions of the basic equations: boundary conditions, initial conditions, and mixed conditions.
- ❺ They will be able to solve these partial differential equations with appropriate boundary conditions on simple domains by the method of separation of variables.
- ❻ They will become familiar with basic facts of Fourier series, sine series, and cosine series, and be able to utilize them in solving initial and boundary value problems.
- ❼ Students will understand and be able to contrast the fundamental properties of these basic partial differential equations.

Assessment Method: Embedded questions throughout homework, quizzes and exams.

Illustration #1: Carefully derive the equation of a string in a medium in which the resistance is proportional to the velocity. ❶

Illustration #2: Interpret the physical significance of the homogeneous Neumann boundary condition for the one-dimensional heat equation. ❶❹

Illustration #3: Classify each of the equations. (A list of several second-order equations is given.) ❷

Illustration #4: Solve the first-order equation $2u_t + 3u_x = 0$ with the auxiliary condition $u = \sin x$ when $t = 0$. ❸

Illustration #5: Solve the boundary value problem for the Laplace equation on the rectangle with non-homogeneous boundary condition. ❺❻

Illustration #6: Show that there is no maximum principle for the wave equation. ❼

Illustration #7: Solve the initial value problem for the one-dimensional wave equation for general initial data. ❹❺

Appendix 3
General Education and Service Courses

In this appendix, in a similar fashion to the previous, we append the descriptors of the general education and service courses the department offers.

The new pre-baccalaureate courses Math 7, Basic Intermediate Algebra and Math 11, Enhance Intermediate Algebra are used in the prerequisite listings.

The main function of a course is described as either General Education (GE), or Service (S) to a specific major or majors. The on listed first is considered more important.

The courses are:

	Main Function	Clientele	Prerequisites
Math 103	GE	COTA, CLA	ELM or Math 7
Math 112	GE & S	CLA, CHHS, CNSM	ELM or Math 11
Math 114	S & GE	CBA	ELM or Math 7
Math 115	S & GE	CBA	ELM or Math 11
Math 119A	S & GE	CNMS	ELM or Math 11
Math 180	GE	CLA, CHHS	ELM or Math 7
MTED 110	S &GE	CED	ELM or Math 7
NEW GE COURSES These courses have been applied to be GE classes for fall 2007			
Math 101	GE & S	CLA, COTA, CE, CNSM	ELM or Math 7
Math 109	GE	CLA, CHHS	ELM or Math 7
Math 113	S & GE	CNSM, COE, CBA	ELM or Math 11

The initials stand for the colleges: COTA, College of the Arts, CBA, College of Business Administration, CLA, College of Liberal Arts, CED, College of education, COE, College of Engineering, CHHS, College of Health & Human Services, CNSM, College of Natural Sciences & Mathematics

In addition Math 122 and 123 are extensively used as service courses to engineers, some economists and scientists. Those descriptors are in the previous appendix.

112

Goals:

¬ Students will formulate real world situations in meaningful mathematical forms, including graphs, tables, diagrams or equations.

\- Students will execute mathematical manipulation and computation in order to solve a posed problem.

® Students will recognize, have knowledge of, be able to combine and evaluate fundamental mathematical expressions and functions such as polynomials and exponentials.

¯ Students will exhibit familiarity and ease of use of basic geometric and arithmetical facts such as the Pythagorean Theorem, similar triangles and ratios.

° Students will interpret the mathematical result about real world situations derived mathematically.

± Students will be able to use counting techniques to compute elementary probability estimates.

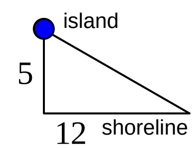
Assessment Method: Embedded questions throughout homework, quizzes and exams.

Illustration #1: Ornithologists have determined that some species of birds tend to avoid flights over large bodies of water during daylight hours, because air generally rises over land and falls over water in the daytime, so flying over water requires more energy.

A bird is released on an island 5 miles from shore. The nesting area is 12 miles down the straight shore from the point on the shore directly opposite the island. The bird uses 10 kcal/mi to fly over land, while it uses 14 kcal/mi to fly over water. Consider the following questions:

¡ How much energy will the bird use if it flies directly from the island to the nesting area?

What is expected is that the student will visualize the information in the form of a triangle ¬:



Thus, by letting d denote the distance from the island to the nesting area, the student would arrive, by the use of the Pythagorean Theorem ¯, to

$$d^2 = 5^2 + 12^2 = 25 + 144 = 169,$$

and so one would conclude that $d = 13$ miles. -

Naturally, the student should continue to answering the question:

The bird will need $13 \times 14 =$ *182 kcal to accomplish that trip.* °

¢ If the bird flies directly over the water into land and then flies over land to the nesting ground, how much energy will the bird need then?

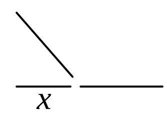
Continuing with the triangle model, the answer is readily arrived at ¬, ®, ° :

$$5\times 14+12\times 10=70+120=190 \text{ kcals.}$$

Now the more sophisticated question:

® Suppose the bird is to fly directly over the water to some point on the shore between the nearest point and the nesting place, and then fly over land to the nesting place. Can the bird save energy by doing this? If so can the bird just use 170 kcals? If so where should the bird fly?

The student is then expected to come up with a variable, x, ¬, - which could represent the distance between the point on shore nearest the island and the point on the shore that the bird will fly to, so the picture now looks like

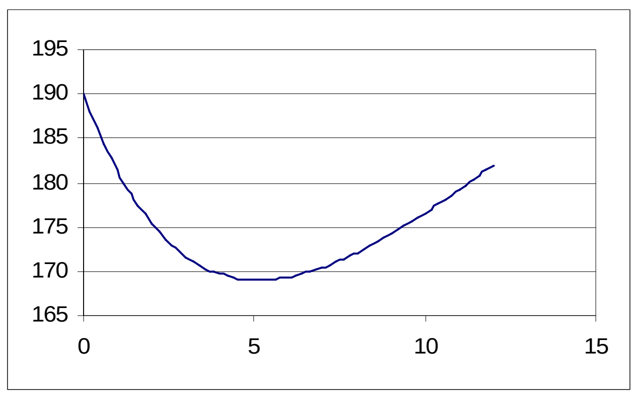


Now the student is expected to use the Pythagorean Theorem once again, and express the distance y that the bird is flying over water as a function of x:

$y^2=5^2+x^2$, so $y=\sqrt{25+x^2}$ ®, ¯. Additionally, the distance the bird is flying over land, $z=12-x$. Thus the energy E that the bird will consume when it flies toward the point at x is given by the function:

$$E(x)=14\sqrt{25+x^2}+10(12-x) \text{ ¬, ®.}$$

The student could get a graphical representation of this function: and see a ready answer for the first part of the question ¬, ®, ° :



Indeed, the bird can spend less energy ° if it flies to some point on shore different from the nearest point and then along the shore.

Also from the picture, the student can identify two destinations that the bird could fly to in order to use exactly 170 kcals. What is needed now is to solve the equation

$$E(x)=14\sqrt{25+x^2}+10(12-x)=170 \text{ - , ®.}$$

Simplifying, it becomes

$14\sqrt{25+x^2}=170-120+10x=50+10x$; or $7\sqrt{25+x^2}=25+5x$. -

Squaring both sides,

$49(25+x^2)=(25+5x)^2=625+250x+25x^2$. -

Simplifying one more time,

$24x^2 - 250x + 600 = 0$. -
Using the quadratic formula,

$$x = \frac{250 \pm \sqrt{4900}}{48} = \frac{250 \pm 70}{48} = \frac{320}{48} \text{ or } \frac{180}{48} = \frac{20}{3} \text{ or } \frac{15}{4}$$ - ,

and finally the student should conclude that that the bird should toward a point that is either $6\frac{2}{3}$ miles or $3\frac{3}{4}$ miles away ° along the shore from the nearest point to the island.

Ultimately, you would like the student to understand that if the bird flew to any point between the two points obtained in the previous problem, the bird would be using less than 170 kcals.

Those students with an inquiring mind, might develop the curiosity of what is the optimal solution for the bird and how few kcals will it need to arrive at the sanctuary.

Illustration #2: A poker hand, consisting of 5 cards, is dealt from a standard deck of 52 cards. Find the probability that the five cards are in the same suit.

The first issue is that the student will realize that the computation of the denominator of the fraction that will give the probability is of utmost importance, ¬, and that that denominator is $C(52,5) = 2,598,960$ ± the number of ways of choosing 5 objects out of 52 objects. Second, the student will compute the numerator of that fraction as $4 \times C(13,5) = 4 \times 1287 = 5,148$ ±, the 4 coming from the number of choices for the suit, and $C(13,5)$ as the number of ways of choosing 5 cards from the 13 cards in that suit. Taking the ratio, the student will give $\frac{5148}{2598960} = .001980$, or approximately 0.2 % as the probability that the five cards are in the same suit ° .

Alternatively, the student could think of a different model and suggest computing the probability that the five cards are hearts, and then multiplying the answer by 4 ¬. Thus the first card being a heart has probability $\frac{13}{52}$ ±, and then the probability that the second card is a heart, given that the first one was, is $\frac{12}{51}$ ±, and then the third one, $\frac{11}{50}$ ±, and the fourth card, $\frac{10}{49}$, and the final card being a heart: $\frac{9}{48}$. The answer should then be:

$$4 \times \frac{13}{52} \times \frac{12}{51} \times \frac{11}{50} \times \frac{10}{49} \times \frac{9}{48} = 0.001980$$ ±, ° .

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Goals:

¬ Students will understand matrices as collectors of information, execute key operations involving matrices, and use these operations to solve relevant problems.

- Students will translate real world situations into systems of linear equations, by identifying the unknowns and establishing relations among them. Students will use Gaussian Elimination to find and describe the meaningful solutions to the problem.

® Students will be able to use counting techniques to compute probability estimates.

¯ Students will use more sophisticated methods in probability theory such as Random Variables, Bayes' Theorem, Expectation and the Binomial Distribution to solve deeper probability problems.

° Students will be familiar with the Central Limit Theorem and how to use it in order to do estimation of probabilities.

± Students will, at all times, be able to interpret the mathematical result about real world situations derived mathematically.

Assessment Method: Embedded questions throughout homework, quizzes and exams.

Illustration #1: David is an independent supplier of restaurant supplies. He needs to supply three different customers with vegetables and with meat. The first customer, Mr. Smith requires 20 crates of vegetables and 50 pounds of meat, the second, Ms. Jones, requires 18 crates of vegetables and 60 pounds of meat, while the third one, Mr. Colt requires 30 crates of vegetable and 30 pounds of meat. David can buy the supplies at 4 different wholesalers, Pavilions (P), Vons (V), Albertson's (A) and Ralph's (R). Pavilions charges $12 for a crate of vegetables and $45 for pound of meat. Similarly, Vons charges $15 and $40 respectively for vegetables and meat, Albertson's prices are $18 and $37 for vegetables and meat while those of Ralph's are $26 and $34 respectively. For each of the three orders, David is going to order both vegetables and meat from only one of the four suppliers (in order to get free delivery). How should David order his supplies?

The student should first store the required purchases in a 2×3 matrix where the rows stand for vegetables and meat respectively and the columns for Mr. Smith, Ms. Jones and Mr. Colt respectively: $\begin{matrix} 20 & 18 & 30 \\ 50 & 60 & 30 \end{matrix}$ ¬. Similarly, then information on the prices should be collected in a 4×2 where the rows stand for Pavilions, Vons, Albertson's and Ralph's and the columns are vegetables and meat respectively: $\begin{matrix} 12 & 45 \\ 15 & 40 \\ 18 & 37 \\ 26 & 34 \end{matrix}$ ¬. Finally, using matrix multiplication $\begin{matrix} 12 & 45 \\ 15 & 40 \\ 18 & 37 \\ 26 & 34 \end{matrix} \quad \begin{matrix} 20 & 18 & 30 \\ 50 & 60 & 30 \end{matrix}$

to obtain the matrix $\begin{matrix} 2490 & 2916 & 1710 \\ 2300 & 2670 & 1650 \\ 2210 & 2544 & 1650 \\ 2220 & 2508 & 1800 \end{matrix}$ ¬, the student should conclude that for Mr. Smith the groceries should come from Albertson's, for Ms. Jones from Ralph's and for Mr. Colt from either Von's or Albertson's ¬, ±.

Illustration #2: One rooster is worth five dollars; one hen is worth three dollars; while three young chicks are worth one dollar. Buying 100 fowls with 100 dollars, how many roosters, hens and chicks?

If we let R stand for the number of roosters, H for the number of hens and C for the number of chickens, then the conditions of the problem easily translate to the following two equations - :

$$R + H + C = 100$$
$$5R + 3H + \tfrac{1}{3}C = 100$$

Or in augmented matrix notation this becomes $\begin{pmatrix} 1 & 1 & 1 & 100 \\ 5 & 3 & \frac{1}{3} & 100 \end{pmatrix}$ ¬. This matrix reduces - to $\begin{pmatrix} 1 & 0 & \frac{-4}{3} & -100 \\ 0 & 1 & \frac{7}{3} & 200 \end{pmatrix}$ so we have that $R = \frac{4}{3}C - 100$ and $H = 200 - \frac{7}{3}C$. In order for the solutions to make sense, we need to have C to be a multiple of 3, and also since $R \geq 0$ and $H \geq 0$, we must have $C \geq 75$ and $C \leq 85$, so the only possibilities for C are 75, 78, 81 and 84. In fact, the solutions are - , ±

R	H	C
12	4	84
8	11	81
4	18	78
0	25	75

Illustration #3: Five people: Mr. A, Ms. B, Mrs. C, Mr. D and Mr. E will stand side by side to pose for a picture. What is the probability that Mr. D and Mr. E will not be standing next to each other?

The student should immediately realize that the number of ways that the five people can pose for a picture is $5! = 120$ ®. The student should also realize that it is easier to count the number of ways Mr. D and Mr. E can stand together, and hence use the complement of what is wanted ¯. So one is reduced to counting the number of ways of arranging A, B, C, D and E for a picture so that D and E stand together. First decision is making D and E stand together, one has two options for it: DE and ED. After that one has to arrange 4 elements: A, B, C, and DE, for which there are 24 ways of doing it, so in total one has $2 \times 24 = 48$ ®

ways. So there are $120 - 48 = 72$ ® ways to arrange A, B, C, D and E for a picture so that D and E do not stand together, and hence the probability is $\frac{72}{120} = \frac{3}{5} = 60\%$ ¯, ±.

Illustration #4: A university claims that 85% of its students graduate. One is to test their veracity by setting up a test of their claim. One selects 12 students at random, and sees how many of them graduate. The decision is to accept the school's claim if at least 8 of the 12 students graduated. What is the probability that one comes to the wrong conclusion if indeed the university's claim is true?

Let *Y* denote the number of students among the 12 that graduated, so *Y* is a binomial random variable with $n = 12$ and $p = .85$ ¯. Thus one will be wrong if one encounters $Y \le 7$, thus one needs to compute $\mathbf{P}(Y \le 7) = 1 - \mathbf{P}(Y \ge 8)$, and the relevant values are:

8	9	10	11	12
0.068284	0.171976	0.292358	0.301218	0.142242

with a sum of 0.976078, and so one will be wrong 2.39% of the time, a truly negligible possibility ¯, ±.

Illustration #4: A company manufactures perfume sprayers. They consider that 5% of their production is defective. A random sample of 600 sprayers is tested, which is large enough to treat as a normal approximation to the binomial °, ¯. In that sample one expects 30 defective ones ¯.

One also has that the standard deviation $\sigma = \sqrt{600 \cdot .05 \cdot .95} \approx 5.34$ ° atomizers. What is the probability then of each of the following?

¡ At least 35 defectives? Since one is $\frac{5}{\sigma} = .936$ above the mean, the probability is 17.62% by inspection of the normal table °, ±.

¢ Between 25 and 35 defectives? Easily, 64.76% °, ±.

£ Suppose one has obtained 50 defectives in the sample—should one worry that perhaps the defectives amount to more than 5%? Since the probability of being 3.7463 standard deviations above the mean is only .02%, one can consider the production to be more defective than was claimed °, ±.

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Goals: Students will:

¬ Exhibit familiarity and ease of use of basic geometric and arithmetical facts such as the Pythagorean Theorem, similar triangles and ratios.

- Demonstrate knowledge and understanding of the trigonometric functions, their interrelations and their relations to triangles and the unit circle.

® Use the trigonometric functions and their inverses, their manipulation and computation, to solve real world problems involving angles and triangles.

_ Model a variety of periodic phenomena using the trigonometric functions and their graphs.

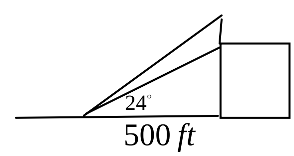
° Interpret the mathematical result about real world situations derived mathematically.

± Exhibit understanding of the arithmetic and geometry of the complex numbers.

Assessment Method: Embedded questions throughout homework, quizzes and exams.

Illustration #1: From a point on the ground 500 ft from the base of a building, it is observed that the angle of elevation to the top of the building is 24° and the angle of elevation to the top of a flagpole atop the building is 27°. Find the height of the building and the length of the flagpole.

What is expected is that the student will visualize the information in the form of two triangles ¬:



Then the student would promptly recognize that $\tan 24^\circ = \frac{h}{500}$ where h denotes the height of the building - , ®, so straightforward computation gives

$$h = 500 \times \tan 24^\circ \approx 500 \times .4452 \approx 223 \text{ feet.}$$

Similarly, if k denotes the height of the building and the flagpole together, we have that - , ®, $\tan 27^\circ = \frac{k}{500}$, so

$$k = 500 \times \tan 27^\circ \approx 500 \times .5095 \approx 255 \text{ feet.}$$

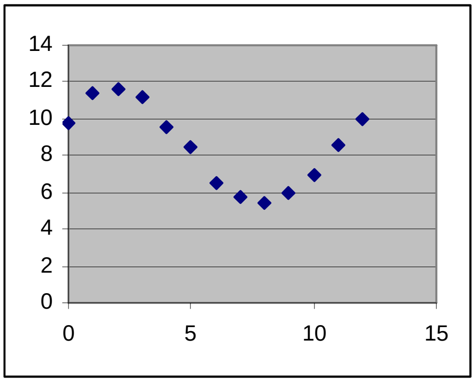
and finally the height of the flagpole is $255 - 223 = 32$ feet. °

Illustration #2: The water depth in a narrow channel varies with the tides. Here is the data for one half day:

Time	12	1	2	3	4	5	6	7	8	9	10	11	12
Depth (ft)	9.8	11.4	11.6	11.2	9.6	8.5	6.5	5.7	5.4	6.0	7.0	8.6	10.0

Then the student is expected to answer questions like the following:

¡ Make a scatter plot of the water depth data where the $x-$axis is time.



And the student should be thinking ahead that a periodic phenomenon (and hence a trigonometric function) is likely to be involved. ¬, ¯

So the second question should be

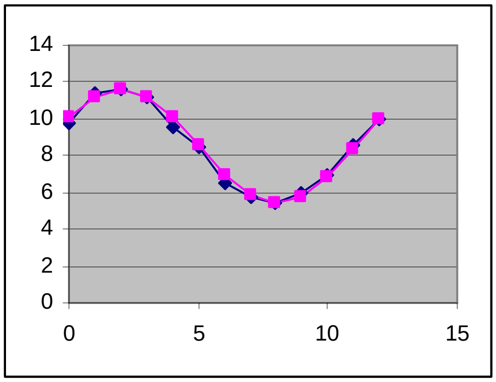
¢ Find a function that models the water depth as a function of time.

The thinking student would select some form of the cosine function (the sine function would do as well) to model it, so if we let $d(t)$ denote the depth of the water measured in feet as a function of time (measured in hours), the student should select an expression of the form:

$$d(t) = a\cos(\omega(t-c)) + b$$

where there are four parameters to be computed, a, b, c and w. The easiest one is b, known as the vertical shift. It represents the average between highest value and lowest value of the curve, so $b = \frac{11.6+5.4}{2} = 8.5$ ®, ¯. The next is the amplitude, a, which is half of the difference between the highest and lowest value: $a = \frac{11.6-5.4}{2} = 3.1$ ®, ¯. The period is 12 hours since the difference between the highest tide and lowest tide is 6 hours, so $\frac{2\pi}{\omega} = 12$, and so ω satisfies $\omega = \frac{2\pi}{12} \approx .52$ ®, ¯. Finally, the phase shift, c, is simply the time of the highest depth, so $c = 2$, and we can model using:

$$d(t) = 3.1\cos(.52(t-2)) + 8.5$$ ®, ¯

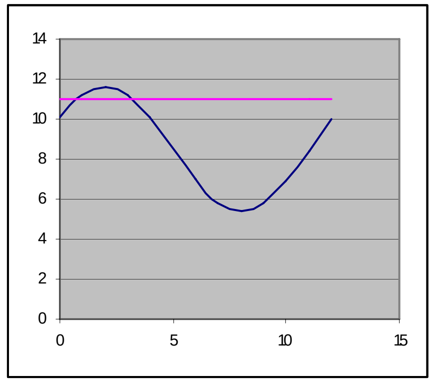


and the student observes graphically that the match is acceptable.

Finally, a third and most interesting question would be:

£ If a boat needs at least 11 ft of water to cross the channel, during which times can it safely do so?

Now the student needs to solve $d(t) = 3.1\cos(.52(t-2)) + 8.5 = 11$, which leads to $\cos(.52(t-2)) = .8065$, and so $t = \dfrac{\arccos(.8065)}{.52} + 2$, $t \approx 3.2$. Since that is the solution in the first quadrant, the other solution is in the fourth quadrant and is given by $t = \dfrac{-\arccos(.8065)}{.52} + 2$, so $t \approx .8$. Transforming these measurements from hours to minutes, one get that the boat should safely travel between 12:48 and 3:12. °



109

Goals: Students will:

¬ Formulate real world situations in meaningful mathematical forms, including graphs, tables, diagrams or equations, and in words.

\- Execute mathematical manipulation and computation in order to solve a posed problem.

® Recognize, have knowledge of, be able to combine and evaluate fundamental mathematical expressions and functions such as polynomials and exponentials.

_ Exhibit familiarity and ease of use of basic geometric and arithmetical facts such as the Pythagorean Theorem, similar triangles and ratios.

° Interpret the mathematical result about real world situations derived mathematically.

Assessment Method: Embedded questions throughout homework, quizzes and exams.

Illustration #1: The Food and Drug Administration labels suntan products with a sun protection factor (SPF) typically between 2 and 45. Multiplying the SPF by the number of unprotected minutes you can stay in the sun without burning, you are supposed to get the increased number of safe sun minutes. For example, if you can stay unprotected in the sun for 30 minutes without burning, and you apply a product with SPF of 10, then supposedly you can sun safely for 30x10=300 minutes, or 5 hours.

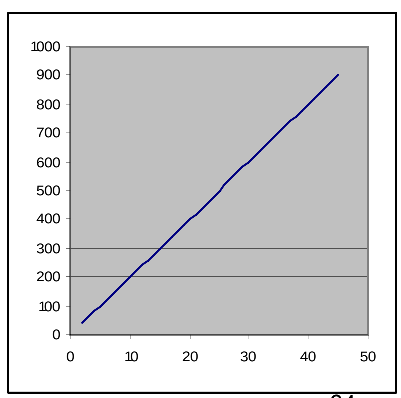
Assume that you can stay unprotected in the sun for 20 minutes without burning.

¡ Give an equation that gives the maximum safe sun time T as a function of the sun protection factor S.

The student is expected to answer ¬: $T = 20 \times S$.

¢ Graph your equation. What is the suggested domain for S?

Using either a calculator or simply a hand-sketch, the student should provide the graph ¬:



And also the student explain that the domain consists of the real numbers between 2 and 45, or equivalently, the close interval $[2,45]$ ®.

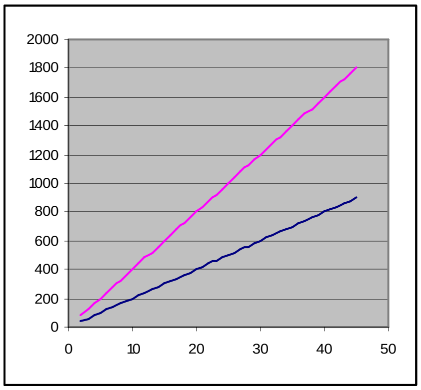
£ Write an inequality that suggests times that would be unsafe to stay out in the sun.

The students should identify that areas above the graph are unsafe, and so their answer should be $T > 20S$ as unsafe ¬ , ° .

¤ Suppose one is using the best product around, how many hours can one safely stay out?

The student should realize that 900 minutes is the highest value that the function achieves ° , and answer “No more than 15 hours”.

¥ How would the graph change if you could stay unprotected for 40 minutes?



Now the relation between the time and the safety level should given by $T = 40 \times S$, and so the accompanying graph is given in contrast as in the picture.

Perhaps the better student will realize that the slope of the line is of some importance.

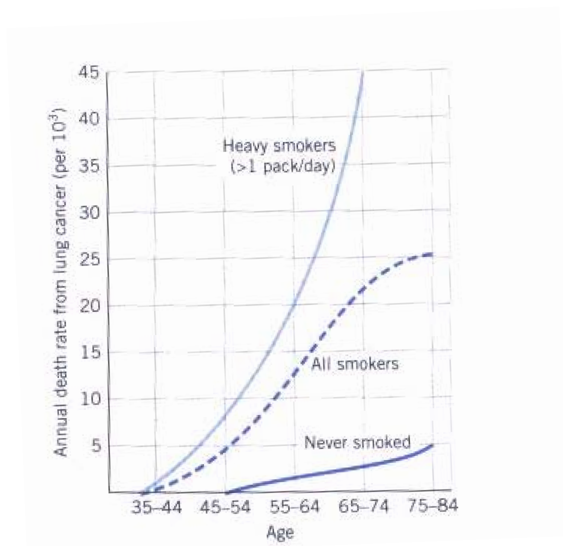


Illustration #2: According to Rubin and Farber’s Pathology, “death from cancer of the lung, more than 85% of which is attributed to cigarette smoking, is today the single most common cancer death in both men and women in the United States.”

The accompanying graph shows the annual death rate (per thousands) from lung cancer for smokers and nonsmokers.

¡ The death rate for nonsmokers is roughly a linear function of age. After replacing each range of ages with a reasonable middle age (for example, you could use 60 to approximate the range 55-64), estimate the coordinates of two points on the graph of nonsmokers and construct a linear model. Interpret your results.

Although the question is purposely open, the easiest points to arrive at are possibly $(50,0)$ and $(80,5)$, so the linear relationship between $D =$death rate per thousand, and $A =$age in years, is given by $D = mA + b$ where $50m + b = 0$ and $80m + b = 5$ ¬. Solving then for m and b - , ® one gets:

$$D = \tfrac{1}{6} A - \tfrac{25}{3} .$$

One interesting consequence is then to observe that since the slope of the line is $\frac{1}{6}$, one could conclude that every 6 added years of age leads to one more death in 1000 people °.

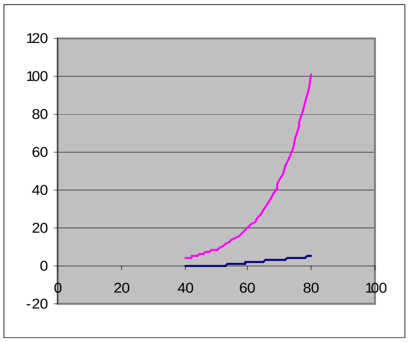
¢ By contrast, those who smoke more than one pack per day show an exponential rise in annual death rate from lung cancer. Estimate the coordinates for those points on the graph for heavy smokers and use the points to construct an exponential model (assume a continuous growth rate). Interpret your results.

Now the student should pursue an expression of the form $D = ce^{mA}$ and the two points that one can use to estimate the parameters $(60, 20)$ and $(70, 45)$. This leads to the equations: $45 = ce^{70m}$ and $20 = ce^{60m}$ - . To solve the equations one needs to look at their quotient ®: $2.25 = e^{10m}$, and solve for m ®:

$$m = \frac{\ln 2.25}{10} \approx 0.08109$$

and then substituting in either expression one gets $c \approx 0.1541$. And one finally obtains $D = .1541e^{.08109A}$. One obvious consequence is that by age 80, the death rate becomes the significant: $D \approx 101$ in every thousand people!

£ Graph the models acquired in ¡ and ¢, and compare with the shapes in the original graph. The graphs are given by



And of course the observation should be made that there is fair resemblance between the original data and the models.

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Goals: Students will:

¬ Formulate real world situations in meaningful mathematical forms, including graphs, tables, diagrams, equations and in words.

- Execute mathematical manipulation and computation in order to solve a posed problem.

® Recognize, have knowledge of, be able to combine and evaluate fundamental mathematical expressions and functions such as polynomials and exponentials.

¯ Exhibit familiarity and ease of use of basic geometric and arithmetical facts such as the Pythagorean Theorem, similar triangles, and ratios.

° Interpret and articulate the mathematical result about real world situations derived mathematically.

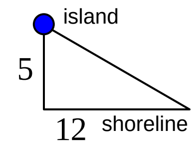
± Exhibit proficiency in understanding and usage of sequence and series terminology and language.

Assessment Method: Embedded questions throughout homework, quizzes and exams.

Illustration: Ornithologists have determined that some species of birds tend to avoid flights over large bodies of water during daylight hours, because air generally rises over land and falls over water in the daytime, so flying over water requires more energy.

A bird is released on an island 5 miles from shore. The nesting area is 12 miles down the straight shore from the point on the shore directly opposite the island. The bird uses 10 kcal/mi to fly over land, while it uses 14 kcal/mi to fly over water. Consider the following questions:

¡ How much energy will the bird use if it flies directly from the island to the nesting area?



What is expected is that the student will visualize the information in the form of a triangle ¬:

Thus, by letting d denote the distance from the island to the nesting area, the student would arrive, by the use of the Pythagorean Theorem ¯, to

$$d^2 = 5^2 + 12^2 = 25 + 144 = 169,$$

and so one would conclude that $d = 13$ miles. -

Naturally, the student should continue to answering the question:

The bird will need $13 \times 14 =$ *182 kcal to accomplish that trip.* °

¢ If the bird flies directly over the water into land and then flies over land to the nesting ground, how much energy will the bird need then?

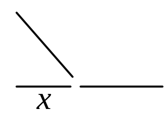
Continuing with the triangle model, the answer is readily arrived at ¬, ®, ° :

$$5\times14+12\times10=70+120=190 \text{ kcals.}$$

Now the more sophisticated question:

£ Suppose the bird is to fly directly over the water to some point on the shore between the nearest point and the nesting place, and then fly over land to the nesting place. Can the bird save energy by doing this? If so can the bird just use 170 kcals? If so where should the bird fly?

The student is then expected to come up with a variable, x, ¬, - which could represent the distance between the point on shore nearest the island and the point on the shore that the bird will fly to, so the picture now looks like

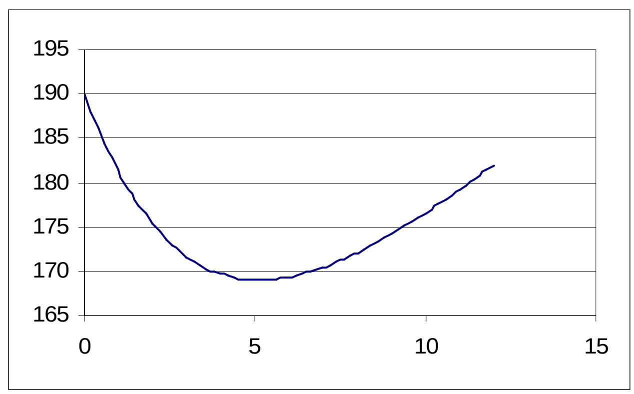


Now the student is expected to use the Pythagorean Theorem once again, and express the distance y that the bird is flying over water as a function of x:

$y^2=5^2+x^2$, so $y=\sqrt{25+x^2}$ ®, ¯. Additionally, the distance the bird is flying over land, $z=12-x$. Thus the energy E that the bird will consume when it flies toward the point at x is given by the function:

$$E(x)=14\sqrt{25+x^2}+10(12-x) \text{ ¬, ®.}$$

The student could get a graphical representation of this function: and see a ready answer for the first part of the question ¬, ®, ° :



Indeed, the bird can spend less energy ° if it flies to some point on shore different from the nearest point and then along the shore.

Also from the picture, the student can identify two destinations that the bird could fly to in order to use exactly 170 kcals. What is needed now is to solve the equation

$$E(x)=14\sqrt{25+x^2}+10(12-x)=170 \text{ - , ®.}$$

Simplifying, it becomes

$14\sqrt{25+x^2}=170-120+10x=50+10x$; or $7\sqrt{25+x^2}=25+5x$. -

Squaring both sides,

$49(25+x^2)=(25+5x)^2=625+250x+25x^2$. -

Simplifying one more time,

$24x^2 - 250x + 600 = 0$. -
Using the quadratic formula,

$$x = \frac{250 \pm \sqrt{4900}}{48} = \frac{250 \pm 70}{48} = \frac{320}{48} \text{ or } \frac{180}{48} = \frac{20}{3} \text{ or } \frac{15}{4}$$ - ,

and finally the student should conclude that that the bird should toward a point that is either $6\frac{2}{3}$ miles or $3\frac{3}{4}$ miles away ° along the shore from the nearest point to the island.

Ultimately, the student should understand that if the bird flew to any point between the two points obtained in the previous problem, the bird would be using less than 170 kcals.

Those students with an inquiring mind, might be curious about the optimal flight plan for the bird and how few kcals will it need to arrive at the sanctuary.

Appendix 4
Letters of Support

Department of Mathematics and Statistics
California State University, Long Beach June 13, 2006

Dr. Robert Mena:

The Center for Student-Athlete Services under the Division of Academic Affairs has worked in partnership with the Athletics Department to academically assist and provide necessary academic support to CSULB's NCAA, Division I student-athletes. To that end, a collaborative effort was established with the Department of Mathematics and Statistics.

The student-athlete population at CSULB is a microcosm of the entire student body and as such, many are admitted to the university with out the requisite math skills to successfully transition through the required General Education and major requirements in math. As the academic success of the student-athlete population is extremely important to CSULB's academic reputation and nationally reported in the media, a connection was made with the Department in an attempt to assist those student-athletes who have not experienced a successful math education in K-12. Adding urgency to this cause is the Chancellor's edict that all remediation must be completed within the first year or the student may not continue at the university. Couple that with NCAA's 2003 Academic Standards that decreased the allotted remediation that can be used for competitive eligibility in the first year from 12 units down to 6; the student-athlete population is being held for more academic requirements than other students on campus.

Meetings have ensued to discuss the needs of this special population and to arrive at appropriate solutions. Discussions on academic support, future changes in math remediation courses, MDP test, and the introduction of online math courses are some of the topics that are continuing to be examined. However, most importantly is the scheduling of math classes at the times conducive to student-athletes. Between team practice, weight training and conditioning, and other expectations on Division I student-athletes, the need to balance the athletic and academic schedule is most difficult for some departments. However, this Department has been very sensitive to the needs of this small but important population has been able to meet the math needs of CSULB's NCAA student-athletes.

The Math and Statistics Department has been most supportive and enthusiastic to assist both the Athletic Department and the Center for Student-Athlete Services in this endeavor. With the added support of the new Athletic Director, Vic Cicles as well as President Alexander's emphasis on "student success", we are grateful and excited about the partnership with the Department of Mathematics and Statistics. If you would like any additional information about our collaborative efforts, please contact me at 562-985-5622 or email at gfenton@csulb.edu.

Sincerely,

Gayle Fenton
Director of Student-Athlete Services
Special Assistant to the Vice Provost for Student Success

Similar letters from other advising sections of the university may be added to the report later.

Appendix 5

Survey of Undergraduate Majors & Graduate Students

Department of Mathematics and Statistics, California State University, Long Beach
Academic Program Review Student Survey, 2006

The results described in this document refer to the Spring 2006 departmental student survey. The survey was conducted between April and May 2006 in a number of upper division and graduate level mathematics and statistics classes. Results were analyzed based on student standing (graduate or undergraduate).

- It is clear that our students are satisfied with the overall performance of the faculty and the department. For example, 74.4% of undergraduates agree or strongly agree that faculty members in the department are interested in their academic development. Moreover, 83.3% of undergraduates sampled agree or strongly agree that faculty are appropriately prepared for the classes they are teaching. On a scale of 1(Strongly disagree) to 5(Strongly agree), the average rating for the 14 questions included in the survey was 3.94 for undergraduates. Satisfaction was even higher with graduate students, where the overall average was 4.16.

In light of the favorable findings, however, there were also a couple of areas that require attention:

- For both undergraduates and graduates, frequency of appropriate course offerings was a concern. 41.4% of undergraduates were neutral or didn't agree that courses were offered enough. The corresponding percentage for graduate students was 45.9%.

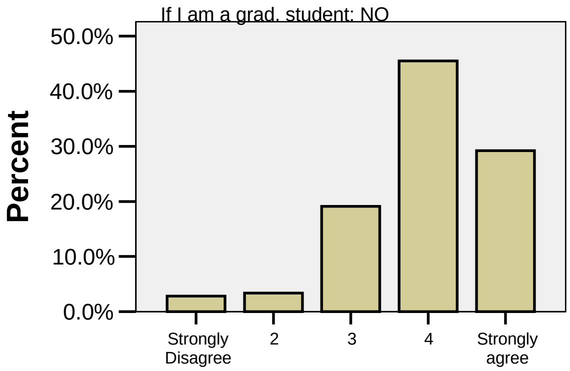
- To the statement that career advisement is available in the department, 38% of undergraduates were neutral, and another 14.5% did not agree. For the same question, 33.3% of graduate students were neutral, and another 8% did not agree.

In summary, Math and Stat majors tend to be satisfied with the education they are receiving and the efforts the department is putting forth in the traditional areas of responsibility. This being said, students are also asking for more variety and availability of courses, and also for more of a legitimate, concerted effort for the department to give them a solid idea of what career possibilities are available for their specific major. Perhaps part of their desire is to see concrete connections between material learned in class and real life problems encountered in work environments.

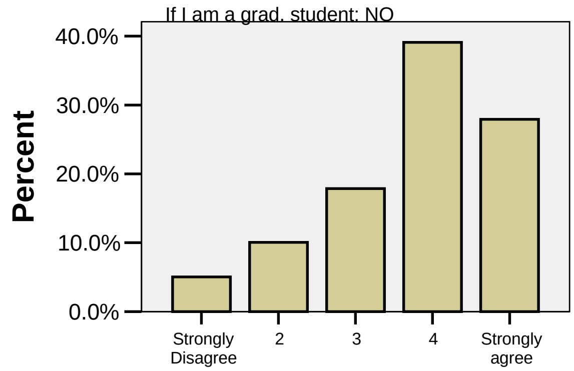
Department of Mathematics and Statistics, California State University, Long Beach
Academic Program Review Undergraduate Student Survey (N=179)

Table

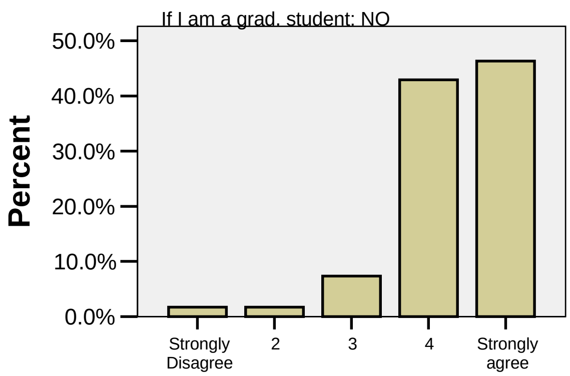
UNDERGRADUATE	*Strongly Disagree*		*2*		*3*		*4*		*Strongly Agree*		*Total*		*Mean*
			n	*%*	*n*	*%*	*n*	*%*			*n*	*%*	*SD*
1. Faculty Members in the department are interested in the academic development of undergraduate majors	5	**2.8**	6	**3.4**	34	**19.0**	81	**45.3**	52	**29.1**	178	**99.4**	3.95 **.934**
2. The undergraduate program of study is academically challenging	3	**1.7**	3	**1.7**	13	**7.3**	76	**42.5**	82	**45.8**	177	**98.9**	4.31 **.817**
3. Faculty in the department are appropriately prepared for their courses	4	**2.2**	5	**2.8**	20	**11.2**	87	**48.6**	63	**35.2**	179	**100.0**	4.12 **.876**
4. I feel the undergraduate program is preparing me for my professional career and/or further study	9	**5.0**	18	**10.1**	32	**17.9**	70	**39.1**	50	**27.9**	179	**100.0**	3.75 **1.121**
5. There is open communication between faculty and undergraduate students about student concerns	6	**3.4**	8	**4.5**	37	**20.7**	68	**38.0**	60	**33.5**	179	**100.0**	3.94 **1.012**
6. Class size is suitable for effective learning	5	**2.8**	3	**1.7**	13	**7.3**	77	**43.0**	81	**45.3**	179	**100.0**	4.26 **.883**
7. Academic advisement is available in the department	6	**3.4**	6	**3.4**	24	**13.4**	68	**38.0**	74	**41.3**	178	**99.4**	4.11 **.991**
8. Career advisement is available in the department	9	**5.0**	17	**9.5**	68	**38.0**	48	**26.8**	35	**19.6**	177	**98.9**	3.47 **1.072**
9. Faculty are available to students outside the classroom	5	**2.8**	3	**1.7**	27	**15.1**	76	**42.5**	68	**38.0**	179	**100.0**	4.11 **.917**
10. Teaching methods used by faculty are effective	4	**2.2**	8	**4.5**	40	**22.3**	90	**50.3**	37	**20.7**	179	**100.0**	3.83 **.886**
11. Procedures used to evaluate student performance are appropriate	3	**1.7**	17	**9.5**	30	**16.8**	100	**55.9**	29	**16.2**	179	**100.0**	3.75 **.897**
12. Frequency of undergraduate major course offerings is satisfactory	12	**6.7**	25	**14.0**	37	**20.7**	77	**43.0**	28	**15.6**	179	**100.0**	3.47 **1.118**
13. Variety of undergraduate major course offerings is satisfactory	6	**3.4**	12	**6.7**	37	**20.7**	84	**46.9**	40	**22.3**	179	**100.0**	3.78 **.979**
14. Degree requirements are clear	6	**3.4**	6	**3.4**	11	**6.1**	64	**35.8**	91	**50.8**	178	**99.4**	4.28 **.968**



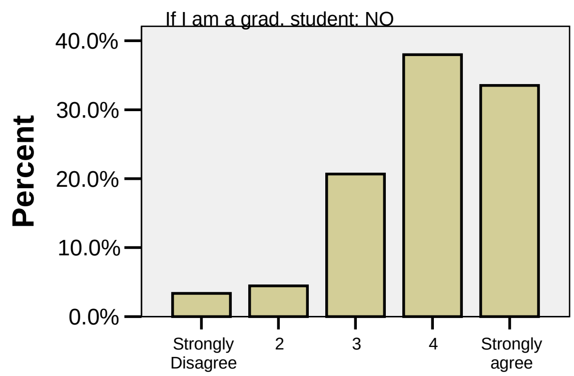
Faculty Members in the Department are interested in the academic development o...



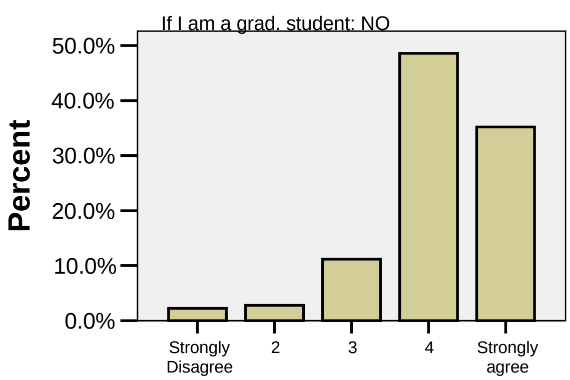
I feel the undergraduate program is preparing me for my professional career...



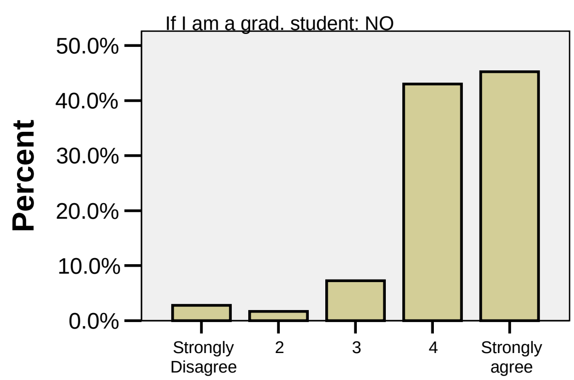
The undergraduate program of study is academically challenging



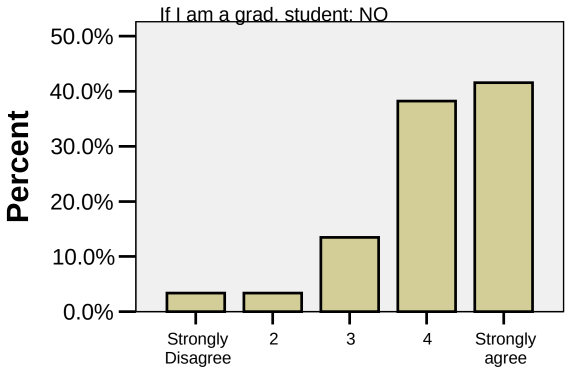
There is open communication between faculty and undergraduate students abou...



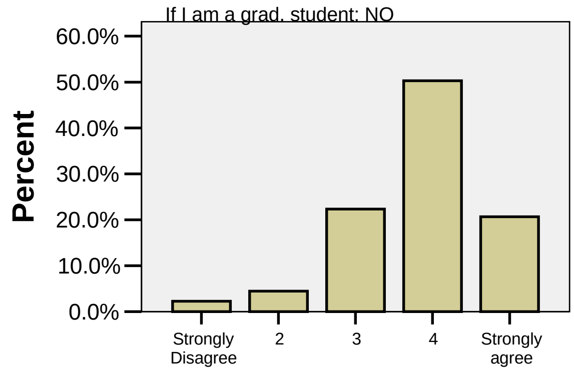
Faculty in the department are appropriately prepared for their courses



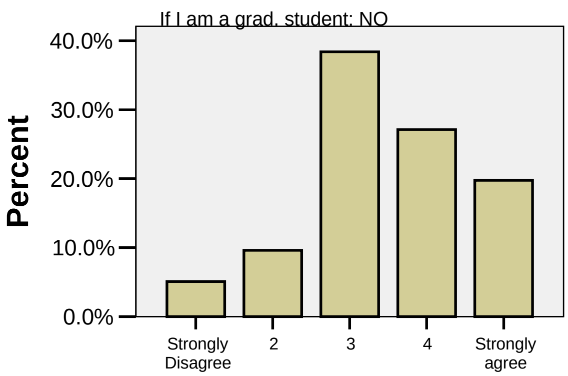
Class size is suitable for effective learning



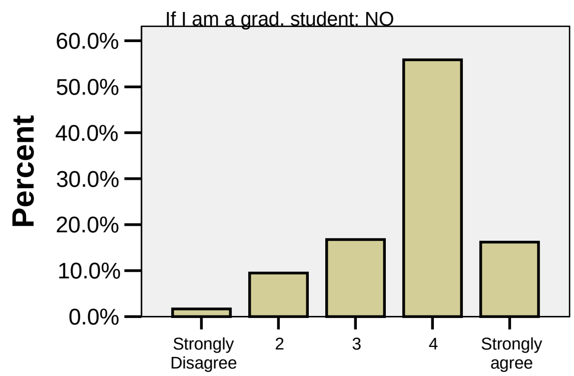
Academic advisement is available in the department



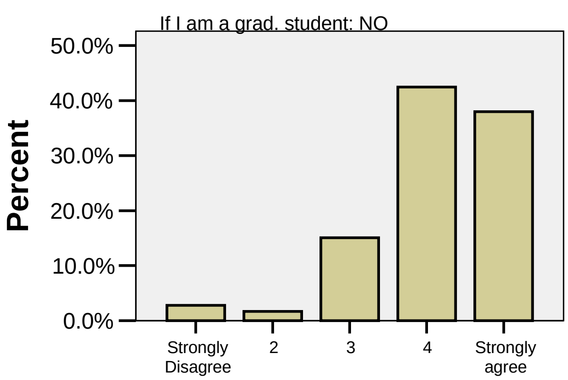
Teaching methods used by faculty are effective



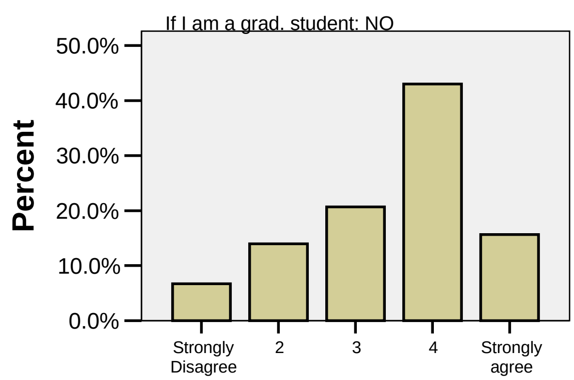
Career advisement is available in the department



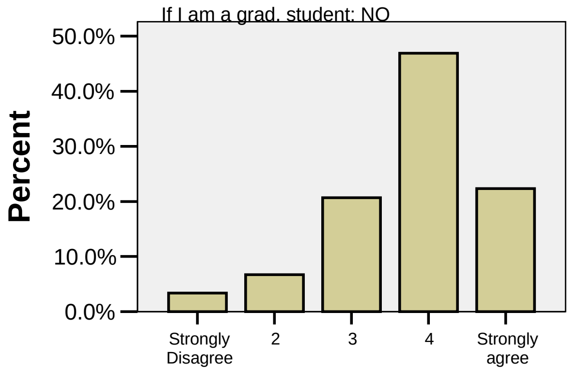
Procedures used to evaluate student performance are appropriate



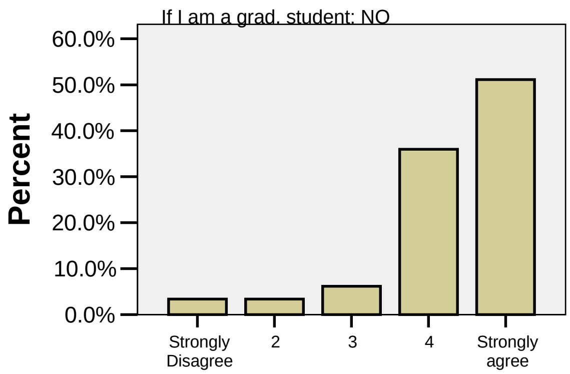
Faculty are available to students outside the classroom



Frequency of undergraduate major course offerings is satisfactory



Variety of undergraduate major course offerings is satisfactory

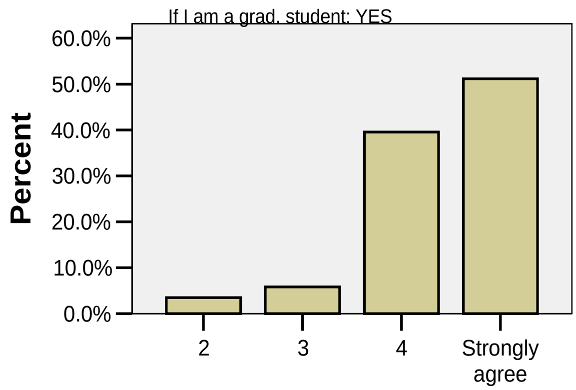


Degree requirements are clear

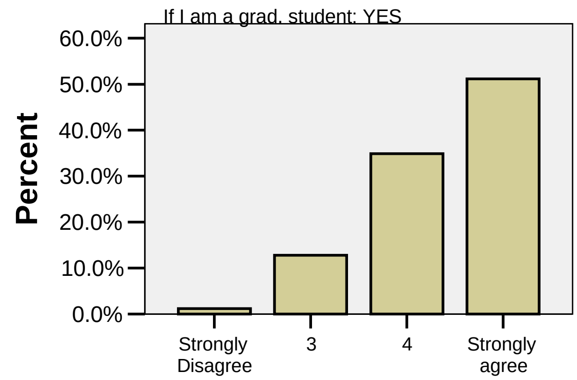
Department of Mathematics and Statistics, California State University, Long Beach
Academic Program Review Graduate Student Survey (N=87)

Table

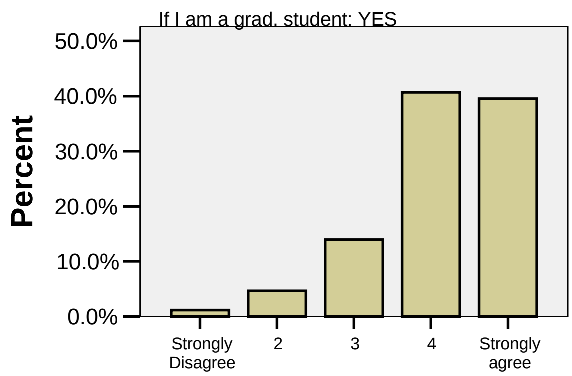
GRADUATE	*Strongly Disagree*		*2*		*3*		*4*		*Strongly Agree*		*Total*		*Mean SD*
			n	*%*	*n*	*%*	*n*	*%*			*n*	*%*	
1. Faculty Members in the department are interested in the academic development of undergraduate majors	0	**0.0**	3	**3.4**	6	**6.9**	34	**39.1**	44	**50.6**	87	**100.0**	4.37 **.764**
2. The undergraduate program of study is academically challenging	1	**1.1**	4	**4.6**	13	**14.9**	35	**40.2**	34	**39.1**	87	**100.0**	4.11 **.908**
3. Faculty in the department are appropriately prepared for their courses	0	**0.0**	3	**3.4**	12	**13.8**	39	**44.8**	33	**37.9**	87	**100.0**	4.17 **.795**
4. I feel the undergraduate program is preparing me for my professional career and/or further study	1	**1.1**	0	**0.0**	12	**13.8**	30	**34.5**	44	**50.6**	87	**100.0**	4.33 **.802**
5. There is open communication between faculty and undergraduate students about student concerns	2	**2.3**	4	**4.6**	11	**12.6**	33	**37.9**	37	**42.5**	87	**100.0**	4.14 **.967**
6. Class size is suitable for effective learning	1	**1.1**	1	**1.1**	3	**3.4**	28	**32.2**	54	**62.1**	87	**100.0**	4.53 **.729**
7. Academic advisement is available in the department	1	**1.1**	1	**1.1**	8	**9.2**	29	**33.3**	48	**55.2**	87	**100.0**	4.40 **.799**
8. Career advisement is available in the department	3	**3.4**	4	**4.6**	29	**33.3**	29	**33.3**	21	**24.1**	86	**98.9**	3.71 **1.004**
9. Faculty are available to students outside the classroom	1	**1.1**	2	**2.3**	5	**5.7**	30	**34.5**	49	**56.3**	87	**100.0**	4.43 **.802**
10. Teaching methods used by faculty are effective	1	**1.1**	2	**2.3**	14	**16.1**	39	**44.8**	31	**35.6**	87	**100.0**	4.11 **.841**
11. Procedures used to evaluate student performance are appropriate	1	**1.1**	2	**2.3**	9	**10.3**	47	**54.0**	28	**32.2**	87	**100.0**	4.14 **.780**
12. Frequency of undergraduate major course offerings is satisfactory	3	**3.4**	14	**16.1**	23	**26.4**	28	**32.2**	19	**21.8**	87	**100.0**	3.53 **1.109**
13. Variety of undergraduate major course offerings is satisfactory	3	**3.4**	6	**6.9**	21	**24.1**	34	**39.1**	23	**26.4**	87	**100.0**	3.78 **1.028**
14. Degree requirements are clear	1	**1.1**	4	**4.6**	2	**2.3**	30	**34.5**	50	**57.5**	87	**100.0**	4.43 **.844**



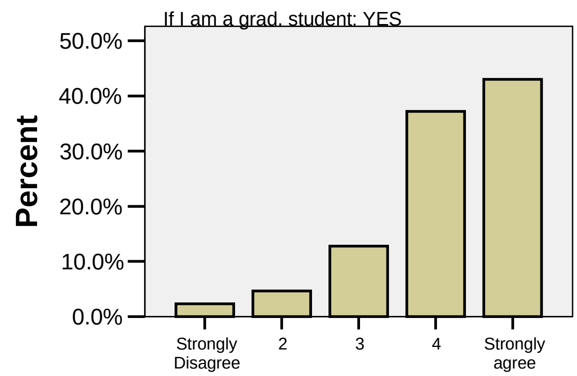
Faculty Members in the Department are interested in the academic development of...



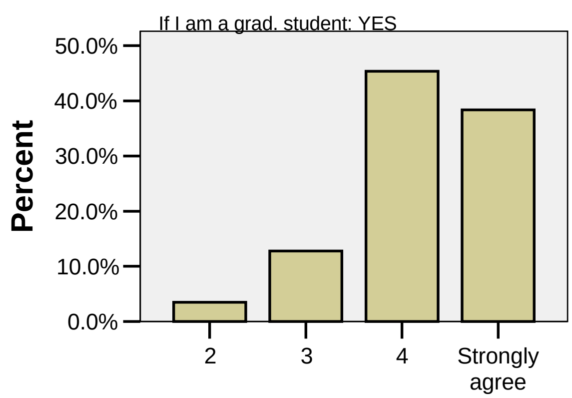
I feel the graduate program is preparing me for my professional career and/or...



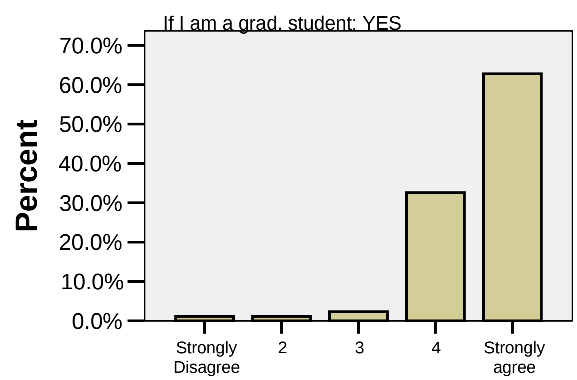
The graduate program of study is academically challenging



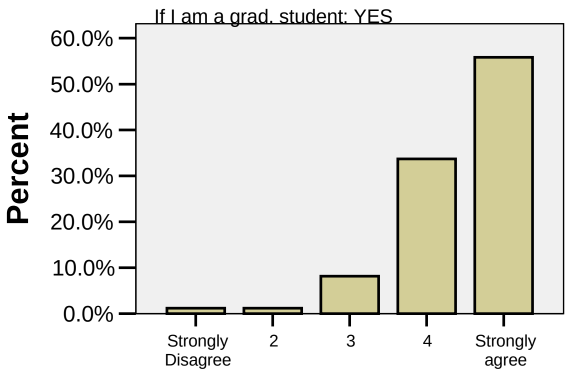
There is open communication between faculty and graduate students about...



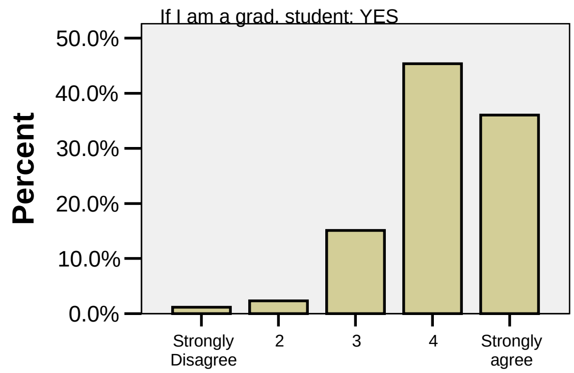
Faculty in the department are appropriately prepared for their courses



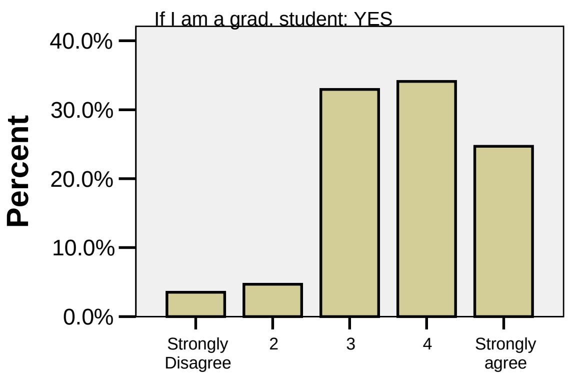
Class size is suitable for effective learning



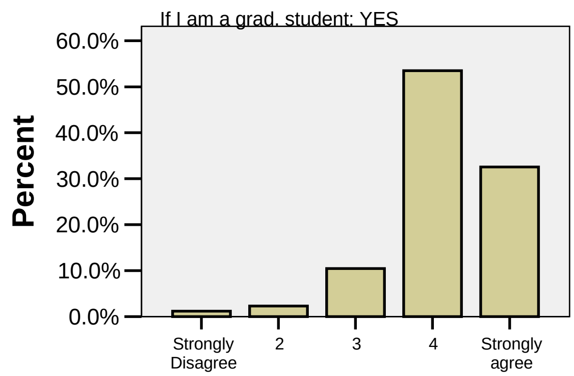
Academic advisement is available in the department



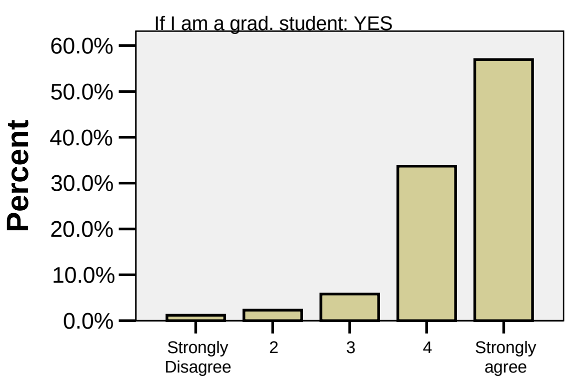
Teaching methods used by faculty are effective



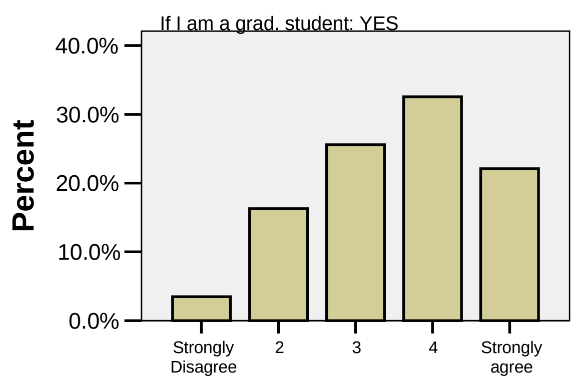
Career advisement is available in the department



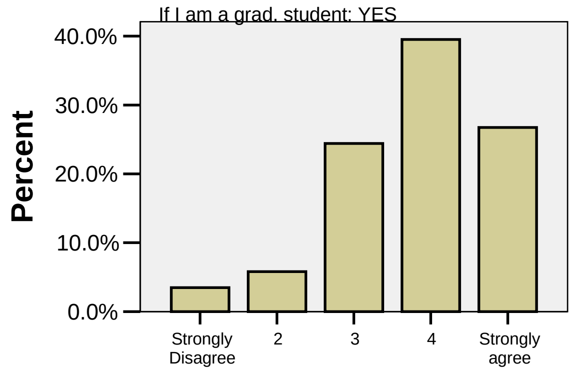
Procedures used to evaluate student performance are appropriate



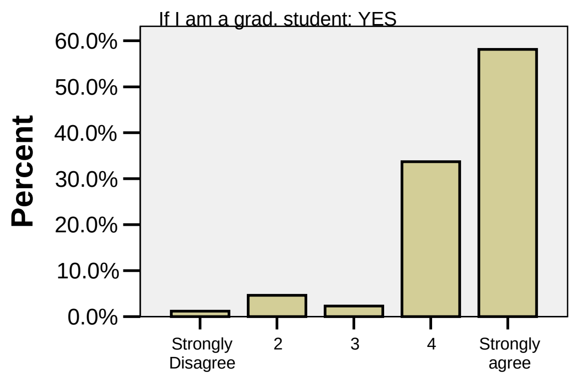
Faculty are available to students outside the classroom



Frequency of graduate major course offerings is satisfactory



Variety of graduate major course offerings is satisfactory



Degree requirements are clear

Appendix 6
Active Faculty from Fall 2001 to Fall 2006

First Name	Last Name	Area	Current Status	Institution Granting Doctorate	Year of Grad.	Year Hired	Tenure Year
Shafqat	Ali	Pure Mathematics	Retired	UC, Santa Barbara	1970	1968	1973
John	Bachar	Pure Mathematics	Retired	UC, Los Angeles	1970	1969	1974
Babette	Benken	Mathematics Education	Assistant Professor	University of Michigan	2004	2006	NA
Joseph	Bennish	Applied Mathematics	Professor	UC, Los Angeles	1987	1988	1993
John	Brevik	Pure Mathematics	Assistant Professor	UC, Berkeley	1996	2006	NA
Linda	Byun	Pure Mathematics	Professor	University of Wisconsin	1984	1984	1990
Bruce	Chaderjian	Applied Mathematics	Associate Professor	UC, Los Angeles	1989	1989	1995
Samuel	Councilman	Pure Mathematics	Professor (FERP)	UC, Los Angeles	1968	1968	1975
Scott	Crass	Pure Mathematics	Associate Professor	UC, San Diego	1996	2001	2005
Linda	DeGuire	Mathematics Education	Professor	University of Georgia	1983	1990	1995
Yu	Ding	Pure Mathematics	Assistant Professor	New York University	2001	2006	NA
Carl	Dorn	Applied Mathematics	Professor	UC, Los Angeles	1967	1968	1982
Morteza	Ebneshahrashoob	Statistics	Professor	Oklahoma State U	1978	1990	1994
Tangan	Gao	Applied Mathematics	Associate Professor	Michigan State University	1999	1999	2005
Y. David	Gau	Pure Mathematics	Professor	Purdue University	1981	1988	1995
Eun Heui	Kim, E. H.	Applied Mathematics	Associate Professor	University of Connecticut	1999	2001	NA
Sung Eun	Kim, S. E.	Statistics	Assistant Professor	UC, Davis	1998	2005	NA
YongHee	Kim-Park	Statistics	Associate Professor	UC, Riverside	1995	1998	2002
Olga	Korosteleva	Statistics	Assistant Professor	Purdue University	2002	2002	NA
Melvin	Lax	Applied Mathematics	Professor	Rensselaer P I	1974	1977	1981
William	Margulies	Applied Mathematics	Professor	Brandeis University	1967	1969	1981

Robert	Mena	Pure Mathematics	Professor	University of Houston	1973	1988	1988
Kent	Merryfield	Pure Mathematics	Professor	University of Chicago	1980	1988	1992
Kathleen	Miller	Pure Mathematics	Resigned	University of Maryland	2001	2001	NA
William	Murray	Pure Mathematics	Assistant Professor	UC, Berkeley	2001	2001	NA
Florence	Newberger	Pure Mathematics	Assistant Professor	University of Maryland	1998	2001	NA
Norma	Noguera	Mathematics Education	Associate Professor	Ohio University	1998	2001	2005
Alan	Safer	Statistics	Associate Professor	University of Wyoming	2000	2000	2005
Howard	Schwartz	Pure Mathematics	Retired	University of Toledo	1969	1969	1973
Angelo	Segalla	Mathematics Education	Professor	UC, Los Angeles	1973	2001	2005
James	Stein	Pure Mathematics	Professor	UC, Berkeley	1967	1989	1991
Kagba	Suaray	Statistics	Assistant Professor	UC, San Diego	2004	2005	NA
Lindsay	Tartre	Mathematics Education	Professor	University of Wisconsin	1984	1985	1993
Robert	Valentini	Pure Mathematics	Professor	Ohio State University	1979	1989	1993
Ngo	Viet	Pure Mathematics	Professor	UC, Berkeley	1984	1989	1993
Derming	Wang	Pure Mathematics	Professor	University of Georgia	1981	1983	1993
Saleem	Watson	Pure Mathematics	Professor	McMaster University	1978	1986	1991
Arthur	Wayman	Pure Mathematics	Professor (FERP)	UC, Los Angeles	1975	1977	1982
Robert	Wilson	Pure Mathematics	Retired	UC, Los Angeles	1974	1966	1975
Wen-Qing	Xu	Applied Mathematics	Associate Professor	New York University	1999	2002	2006
William	Ziemer	Applied Mathematics	Professor	Carnegie Mellon U	1989	1989	1995

Teaching Units Assignments for

Faculty & Part-Time Instructors

Tenured & Tenure-Track	Current Status	F01	S02	F02	S03	F03	S04	F04	S05	F05	S06
Ali	Retired	6	6	6	6	5.6	5.6	5.6	5.6	5.6	5.6
Bachar	Retired	6	6.3	6	6	6	6	6	6	NA	NA
Bennish	Tenured	0	8.6	8.6	8.6	11.6	12.6	11.6	11.6	11.6	11.6
Byun	Tenured	11.6	8.6	8.6	8.6	11.6	0	11.6	11.6	11.6	11.6
Chaderjian	Tenured	10.6	9.6	0	0	9.6	13.6	10.6	12.6	10.6	12.6
Councilman	FERP	0	5.6	5.6	5.6	5.6	5.6	3	3	3	3
Crass	Tenured	6	6.7	11.5	4.8	10.5	10.6	11.6	9	0	0.3
DeGuire	Tenured	9.9	9.9	9.3	9.9	12.9	9.9	12.9	0	13.2	9.9
Dorn	Tenured	11.6	11.6	11.6	11.6	11.6	11.6	11.6	11.6	11.6	11.6
Ebneshahrashoob	Tenured	11.6	9	9.5	9.5	9.5	10.2	12	9	15	9
Gao	Tenured	4.2	3	6	6	9.5	8.6	8.6	11.6	0	8.6
Gau	Tenured	5.6	11.6	8.6	8.6	11.6	11.6	11.6	11.6	11.6	9.5
Kim, E. H.	Track	5.6	6.8	5.6	6	8.6	3	6	5.6	6.3	6.3
Kim, S. E.	Track	NA	NA	NA	NA	NA	NA	NA	NA	6	6.5
Kim-Park	Tenured	9.3	8.6	0	8.9	11.6	11.9	14.6	9	11.6	9.1
Korosteleva	Track	NA	NA	6	6	7.2	7.5	9	9.3	0	10
Lax	Tenured	11.6	11.6	11.6	11.6	11.6	11.6	11.6	11.6	11.6	11.6
Margulies	Tenured	11.6	11.6	11.6	11.6	11.6	11.6	11.6	11.6	11.6	11.6
Mena	Tenured	10	3	12	12	11.6	12	12.1	11.9	3	3.5
Merryfield	Tenured	9.6	9	9.6	8.6	9.6	9	10	9	9.9	9
Miller	Resigned	6.6	6.6	6.6	6.6	NA	NA	NA	NA	NA	NA
Murray	Track	6	6	6	6.5	9.1	9.6	11.6	9.4	8.6	6.6
Newberger	Track	6.5	6	6	7.5	9.9	7	6.8	9	8.9	1
Noguera	Tenured	6.6	6.6	6.6	6.6	7.05	9.4	12.3	9	11.4	8.7
Safer	Tenured	6.5	6	6.3	7.3	9.6	11.5	9.2	8.5	11.5	9
Schwartz	Retired	6	6	6	6	NA	NA	NA	NA	NA	NA
Segalla	Tenured	6.5	6.5	7	7.8	7.85	6.5	9.5	6.5	9.5	9.5
Stein	Tenured	6	0	9	6	11.6	12	11.6	11.6	11.6	11.6
Suaray	Track	NA	NA	NA	NA	NA	NA	NA	NA	6	6
Tartre	Tenured	9.9	9.9	6.6	6.6	8.1	6.6	8.1	9.9	9	13.2
Valentini	Tenured	11.6	8.6	8.6	8.6	11.6	12.1	12.1	12.1	11.6	11.9
Viet	Tenured	8.6	8.6	8.6	8.6	8.6	8.6	11.6	11.6	11.6	5.6
Wang	Tenured	8.6	8.6	8.6	8.6	11.6	11.6	12	8.6	3	11.6
Watson	Tenured	9	8.6	8.6	9	12.1	9	9	9	0	0
Wayman	FERP	2.8	0	0	0	3.5	2	3	4.3	0	6
Wilson	Retired	0	10	0	11.6	9.6	0	10	0	14	NA
Xu	Tenured	NA	NA	5.6	3	6	5.6	9	9	8.6	9.5
Ziemer	Tenured	3	3	5.6	5.6	8.6	11.6	8.6	6	8.6	11.6

Part Time Lecturers	Current Status	F01	S02	F02	S03	F03	S04	F04	S05	F05	S06
Acosta	Currently Employed	14.6	11.6	11.6	11.6	11.6	11.6	12.1	11.6	11.6	11.6
Akins	Currently Employed	6	6	8.6	6	8.6	NA	NA	5.6	8.6	8.6
Araeipour	Currently Employed	14.6	12	12	12	5.6	NA	NA	5.6	11.6	8.6
Arsenidis	Currently Employed	15	15	15	15	15	15	15	15	15	15
Ath	Currently Employed	NA	NA	NA	NA	NA	NA	NA	NA	3	3
Baham	Currently Employed	12.9	12.9	13.2	13.2	13.2	13.2	13.2	14.7	14.7	14.4
Ball		8.9	6.3	12	12	15	15	12	12	15	12
Blair	Past Employee	6	6	3	3	6	NA	NA	NA	NA	NA
Bourouis-Benyassine	Past Employee	12	NA	NA	NA	NA	NA	NA	NA	NA	NA
Brown	Past Employee	NA	NA	14.2	NA	NA	NA	NA	NA	NA	NA
Brownson	Currently Employed	8.6	8.6	8.6	8.6	8.6	8.6	9	9	8.6	8.6
Chaffee	Currently Employed	9	6	9	6	NA	NA	9	9	9	9
Chammas	Past Employee	NA	NA	6	NA	NA	NA	NA	NA	NA	NA
Chen	Past Employee	NA	NA	13.2	NA	NA	NA	NA	NA	NA	NA
Clarke	Past Employee	NA	NA	1.65	1.65	NA	NA	NA	NA	NA	NA
Coopman	Past Employee	NA	9	NA	NA	NA	NA	NA	NA	NA	NA
Della Rocca	Currently Employed	9	6	10.3	10	10.6	9	8.6	9.9	9	13
English	Currently Employed	11.2	11.6	9	8.6	6	5.6	5.6	6	11.2	8.6
Estephan	Past Employee	12	12	9	6.66	NA	NA	NA	NA	NA	NA
Faridpak	Currently Employed	11.6	9	14.6	11.2	14.6	14.2	14.2	14.2	14.6	14.6
Foster	Past Employee	9	6	6	NA	NA	NA	NA	NA	NA	NA
Foti	Past Employee	3	3	3	3	3	3	4.8	3.9	2.4	NA
Geier	Currently Employed	15	15	15	15	15	15	15	15	15	15
Gelenchi	Past Employee	12	6	NA	NA	NA	NA	NA	NA	NA	NA
Ghamsary	Currently Employed	9	12	9	9	9	9	6	6	9	9
Gibson	Currently Employed	12	12	11.6	11.6	12	11.2	11.6	12	14.6	14.6
Granillo	Currently Employed	12.9	12.9	13.2	14.58	13.95	14.7	15.6	14.7	14.7	15
Hamza	Currently Employed	14.2	14.6	11.6	11.2	11.6	11.6	14.2	14.2	14.2	15
Heidt	Past Employee	11.6	6	14.6	11.6	14.6	8.6	14.2	15	14.6	14.2
Hernandez	Currently Employed	14.6	14.6	14.6	14.6	14.6	14.6	14.2	14.2	14.2	14.2
Hoang	Past Employee	12	NA	NA	NA	NA	NA	NA	NA	NA	NA
Hunter	Currently Employed	14.2	14.6	14.6	14.6	14.6	14.6	14.6	NA	14.6	NA
Igolnikov	Currently Employed	14.6	14.2	14.2	11.2	11.2	11.6	14.2	14.2	14.2	12
Jackson	Past Employee	6	NA	NA	NA	NA	NA	NA	NA	6	NA
Khoddam	Currently Employed	6	6	6	6	6	6	6	6	0	6
Komzsik	Past Employee	NA	NA	6	5.6	6	NA	NA	NA	NA	NA
Lamanski	Currently Employed	NA	NA	NA	NA	NA	NA	NA	NA	6	3
Lau	Currently Employed	3	4.5	6	6.66	3.75	4.5	7.5	7.5	7.4	10.1
Leongson	Past Employee	NA	NA	8.6	12.3	12.9	9.6	12.6	13.2	14.6	12.6
Lewandowski	Currently Employed	14.6	15	6	6	6	6	6	6	6	6
Lindgren	Currently Employed	11.6	17.6	*15	NA	14.6	NA	0	0	12	NA
Lopez	Currently Employed	NA	NA	NA	NA	NA	NA	NA	3.3	2.4	4.5
Mariani	Past Employee	6.3	3.3	1.65	1.65	NA	NA	NA	NA	NA	NA
Mbanefo	Currently Employed	11.2	11.2	11.2	11.2	8.6	8.6	8.6	9	9	8.6
McCance	Currently Employed	NA	NA	NA	NA	NA	NA	NA	NA	12	9
McKay	Currently Employed	15.1	14.6	12	12	15	15.3	15	12	12	12
Misajon	Currently Employed	15	15	12	12	12	9	8.6	12	15	9
Moon	Currently Employed	14.2	14.2	11.6	11.6	11.6	11.2	11.6	11.2	11.2	11.6

Nasab	Past Employee	NA	NA	5.6	NA	NA	NA	NA	NA	NA	NA
Nguyen, A.L.	Past Employee	6	NA	NA	NA	NA	NA	NA	NA	NA	NA
Nguyen, S.	Past Employee	6	6	9	6	8.6	NA	NA	NA	NA	NA
Oba	Currently Employed	9.9	11.9	9.9	12.2	13.2	12.9	12.9	12.6	13.2	12.2
Pack	Past Employee	NA	NA	5.6	5.6	NA	NA	NA	NA	NA	NA
Petrie	Currently Employed	9.6	9.6	6.6	6.6	9.6	9.6	9	9.6	9.6	9
Pomerantsev	Currently Employed	14.9	12.9	13.2	13.2	13.2	13.2	13.2	13.2	13.2	13.2
Riley	Past Employee	4.5	5.5	6	3.8	6.81	3.9	1.5	NA	NA	NA
Roeun	Past Employee	9	12	12.3	12.3	12	NA	NA	NA	NA	NA
Rosenberg	Currently Employed	6	6	6	6	6	6	6	6	6	6
Sajjadham	Currently Employed	NA	NA	NA	NA	NA	NA	NA	NA	9	3
Saso	Past Employee	11.6	3	NA	NA	NA	NA	NA	NA	NA	NA
Scully	Past Employee	5.6	NA	NA	NA	NA	NA	NA	NA	NA	NA
Sklar	Past Employee	9.9	0	NA	NA	NA	NA	NA	NA	NA	NA
Smith	Past Employee	NA	5.6	NA	NA	NA	NA	NA	NA	NA	NA
Sobel	Past Employee	0	NA	0	NA	NA	NA	NA	NA	NA	NA
Squires	Past Employee	12	11.2	NA	NA	NA	NA	NA	NA	NA	NA
Sroka	Past Employee	NA	NA	11.2	NA	NA	NA	NA	NA	NA	NA
Stack	Past Employee	11.2	11.2	11.2	5.6	NA	NA	NA	NA	NA	NA
Sterling	Currently Employed	NA	NA	NA	NA	NA	NA	NA	NA	3	6
Swigart	Past Employee	NA	NA	12	NA	NA	NA	NA	NA	NA	NA
Takashima	Past Employee	6	6	3	NA	NA	NA	NA	NA	NA	NA
Tanaka	Past Employee	NA	NA	NA	NA	NA	NA	3	3	3	NA
Tran	Currently Employed	NA	NA	NA	NA	NA	NA	NA	NA	12	6
Waddle	Past Employee	12.3	9.6	NA	NA	NA	NA	NA	NA	NA	NA
Ward		NA	9.9	9.9	9.9	12.2	13.2	9.9	9.9	9.6	0
White	Currently Employed	NA	NA	NA	NA	NA	NA	NA	7.8	6.9	10.8
Wu	Currently Employed	NA	NA	NA	NA	NA	NA	NA	NA	3.3	6.6
Wuth	Past Employee	NA	NA	NA	NA	NA	NA	NA	NA	12	6
Zugates	Currently Employed	5.6	6	12	11.2	8.6	NA	NA	NA	6	NA

∗ Lindgren's D-WTU without the addition of his SAR units totals 15.00, with SAR, it totals 21.0 D-WTU