

CALIFORNIA STATE UNIVERSITY, LONG BEACH

THE MATHEMATICS COLLOQUIUM

Presents

Dr. Marija Rašajski

Assistant Professor
School of Electrical Engineering, University of Belgrade, Serbia, Europe

Visiting Scholar
University of California, Irvine

Speaking on

Multicyclic Reflexive Graphs

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Abstract: For a simple graph G (an undirected graph without loops or multiple edges) let $P_G(\lambda) = \det(\lambda I - A)$ be the characteristic polynomial of its $(0,1)$ adjacency matrix A , also called the *characteristic polynomial* of G . The roots of $P_G(\lambda)$ are the *eigenvalues* of G and, since they are real (A is a real and symmetric matrix), they can be arranged in nonincreasing order: $\lambda_1(G) \geq \lambda_2(G) \geq \dots \geq \lambda_n(G)$. The family of eigenvalues is the *spectrum* of G .

Graphs with the property $\lambda_2 \leq 2$ are called *reflexive graphs*. These graphs correspond to sets of vectors in the Lorentz space $R^{p,1}$ having Gram matrix $2I - A$ (and consequently norm 2 and mutual angles 90° and 120°) and they are Lorentzian counterparts of the spherical and Euclidean graphs, which occur in the theory of reflection groups, having direct application to the construction and the classification of such groups. If $\lambda_2 \leq 2 \leq \lambda_1$, they are also known as *hyperbolic graphs*.

A *cactus* or a *treelike* graph is a graph in which any two cycles have at most one common vertex. θ -*graphs* are bicyclic graphs whose two cycles have a common path.

We search for the reflexive graphs within some classes of *cacti* and θ -*graphs*. Since the property $\lambda_2 \leq 2$ is a *hereditary* one (every induced subgraph preserves this property), the results are expressed through the sets of maximal graphs. We consider classes of maximal reflexive cacti whose cycles do not form a bundle, and classes of maximal reflexive θ -graphs arising from a Smith tree and a cycle attached to it in a specified way. *Smith graphs* are connected graphs with the property $\lambda_1 = 2$. Smith graphs are widely present in the sets of maximal reflexive graphs investigated so far. Many families of such graphs can be described completely or almost completely by Smith graphs.

In this talk I will present some of the results on reflexive cacti and θ -graphs. Also, I will show the important role of the Smith graphs in maximal reflexive graphs.