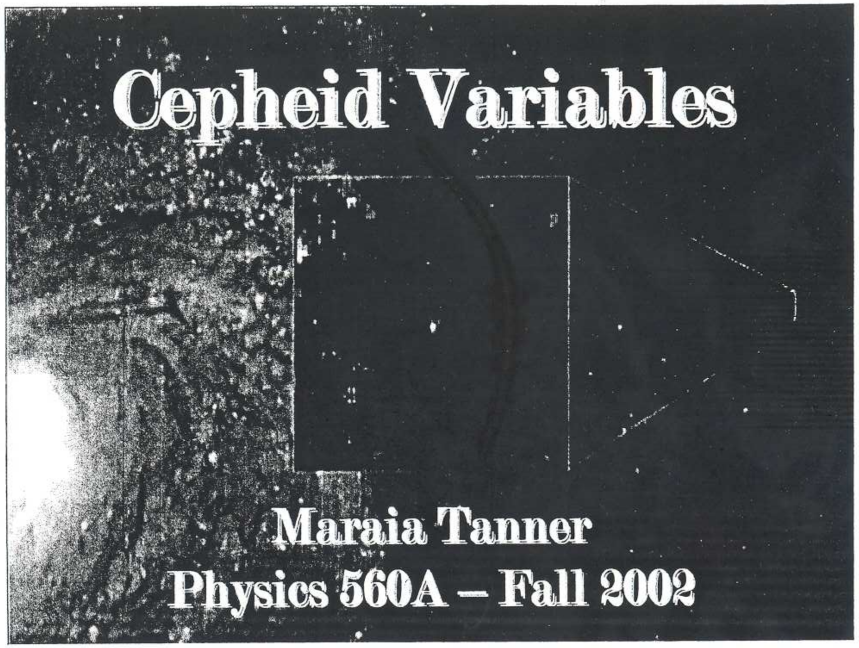

Cepheid Variables

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Physics 560A – Fall 2002

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Discovery of Cepheid Variables

Although the discovery of the first Cepheid Variable is most often accredited to John Goodricke for his work on Delta Cephei in 1784, there were others before him that noticed that the brightness of some stars varied.

In 1596, David Fabricius saw that Mira (aka Omicron Ceti) varied its brightness. Later in 1670, Geminiano Montanari observed that the star Algol's brightness also fluctuated.

Yet it wasn't until 1783 when Goodricke calculated a regular period for Algol (68 hours and 50 minutes) that the study of variable stars truly got underway.

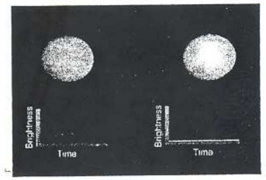
What is a Cepheid Variable?

There are two classifications of variable stars, RR Lyrae and Cepheid Variables. RR Lyrae have approximately a Solar mass and are yellow-white giants with luminosities on the order of 100 times that of the Sun. Cepheid Variables are yellow supergiants with several Solar masses and luminosities on the order of 20,000 times that of the Sun.

These stars pulsate as the result of a special relationship between pressure and gravity. One idea is that as radiation emanates from the star, some of the He^{+} is ionized into He^{+2} leading the surface of the star become more opaque.

As the surface darkens, less energy is able to escape therefore heating the gas within the star. As the gas heats it pushes outward expanding the star's radius. As the star grows in volume, the gas cools allowing the pressure inside to drop (He^{+2} converts back to He^{+}) and gravity to once again dominate by pulling everything inward. The cycle then is able to begin again.

We see the following:



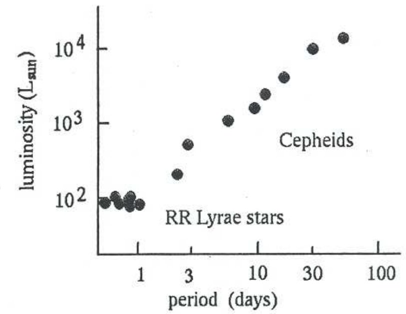
Why are Cepheids important?

Of course there is always interest in anything that is out of the ordinary; however, Cepheid Variables serve a much greater function than merely giving us something unique to look at.

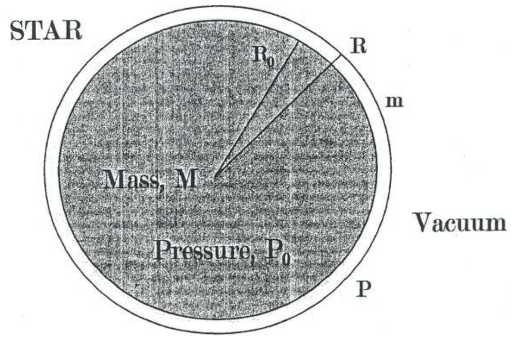
In 1912 Henrietta Leavitt arranged the periods of 25 Cepheid type stars and found there was a direct relationship between the period and the stars luminosity. This discovery lead to following the relationship:

$$M_v = -2.78 \log_{10}\left(\frac{P}{10 days}\right) - 4.13$$

This relationship allows astronomers to determine the distances of far off stars which has lead to more accurate values of the Hubble constant, amongst other similarly important relationships in astronomy.



Simplified Cepheid Model



Period

From Newton's Second Law:

$$(1)\quad m\frac{d^2R}{dt^2} = -\frac{GMm}{R^2} + 4\pi R^2 P$$

In equilibrium, R is constant:

$$(2)\quad -\frac{GMm}{R^2} + 4\pi R^2 P = 0$$

$$(3)\quad \frac{GMm}{R_0^{\,2}} = 4\pi R_0^{\,2} P$$

$$(3a)\quad 4\pi \mathrm{R}_0 P = \frac{GMm}{R_0^{\,3}}$$

Let:

$R = R_0 + \partial R$

$P = P_0 + \partial P$

Substituting back into Equation (1):

$$(4)\quad m\frac{d^2(R_0+\partial R)}{dt^2} = -\frac{GMm}{(R_0+\partial R)^2} + 4\pi(R_0+\partial R)^2(P_0+\partial P)$$

By first order approximation:

$$\frac{1}{(R_0+\partial R)^2} \approx \frac{1}{R_0^{\,2}}\left(1 - 2\frac{\partial R}{R_0}\right)$$

Assume ∂^2 terms negligible, so (4) simplifies to:

$$(5)\quad m\frac{d^2(\partial R)}{dt^2} = -\frac{GMm}{R_0^{\,2}} + \frac{2GMm}{R_0^{\,3}} + 4\pi R_0^{\,2} P_0 + 8\pi R_0 P_0 \partial R + 4\pi R_0^{\,2}\partial P$$

From Equation (2):

$$(6)\quad m\frac{d^2(\partial R)}{dt^2} = \frac{2GMm}{R_0^{\,3}} + 8pR_0P_0\partial R + 4pR_0^{\,2}\partial P$$

If we assume the expansion and contraction of gasses is adiabatic, we get:

$P_0 V_0^{\gamma} = PV^{\gamma}$

$PV^{\gamma} =$ constant

$V = \frac{4}{3}\pi R^3$ for sphere

so $PR^{3\gamma} =$ constant and

$$(7)\quad \frac{\partial P}{P_0} = -3\gamma\frac{\partial R}{R_0}$$

Substituting (3a) and (7) into (6):

$$(8)\quad \frac{d^2(\partial R)}{dt^2} = -(3\gamma - 4)\frac{GM}{R_0^{\,3}}\partial R$$

If $\gamma \succ \frac{4}{3}$, (8) is equation for simple harmonic oscillation given by:

$$(9)\quad \partial R = ASin(\omega t)$$

Putting (9) back into (8) we get:

$$(10)\quad \omega^2 = (3\lambda - 4)\frac{GM}{R_0^{\,3}}$$

The period is given by:

$$(11)\quad T = \frac{2\pi}{\omega}$$

$$(12)\quad T = \frac{2\pi}{\sqrt{(3\gamma - 4)\frac{GM}{R_0^{\,3}}}}$$

Mathematica Findings for Period

$$T = \frac{2\pi}{\sqrt{(3\gamma - 4)\frac{GM}{R^3}}};$$

Tsec = T

520470.

$$Tdays = \frac{Tsec}{86400}$$

6.02395

$\gamma = \frac{5}{3}$; Ideal Gas

$G = 6.67\ 10^{-11}$;

$Rsun = 6.96\ 10^{8}$;

R = 30 * Rsun

2.088×10^{10}

$Msun = 1.989\ 10^{30}$;

M = 10 Msun

1.989×10^{31}

ΔRadii, Velocity and Pressure

Same as before:

(1) $m\frac{d^2R}{dt^2} = -\frac{GMm}{R^2} + 4\pi R^2 P$

(2) $P_0 V_0^{\gamma} = PV^{\gamma}$

$V = \frac{4}{3}\pi R^3$ for sphere

(3) $P = P_0\left(\frac{R_0}{R}\right)^{3\gamma}$

Let $R = R_0 r$, and $\gamma = \frac{5}{3}$ for an ideal gas:

(4) $P = P_0\left(\frac{1}{r}\right)^5$

Then (1) becomes:

(5) $mR_0\frac{d^2r}{dt^2} = -\frac{GMm}{R_0^2 r^2} + 4\pi R_0^2 r^2 P$

(6) $\frac{d^2r}{dt^2} + \frac{GM}{R_0^3 r^2} - \frac{4\pi R_0^2 r^2 P}{m} = 0$

Substituting (4) into (6) we get:

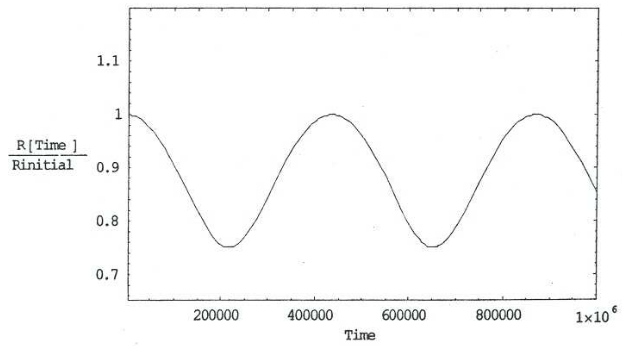
(7) $\frac{d^2r}{dt^2} + \frac{GM}{R_0^3 r^2} - \frac{4\pi R_0^2 P_0}{mr^3} = 0$

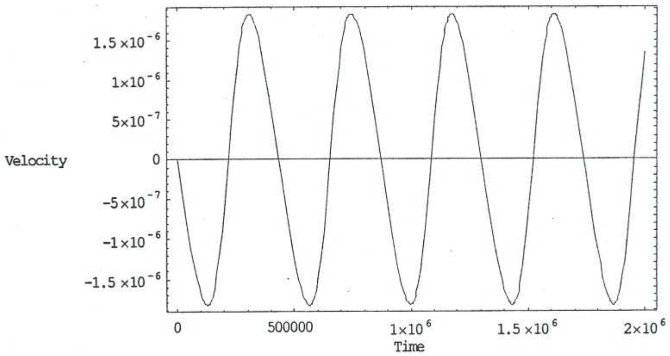
(8) $\frac{d^2r}{dt^2} + \frac{\alpha}{r^2} - \frac{\beta}{r^3} = 0$

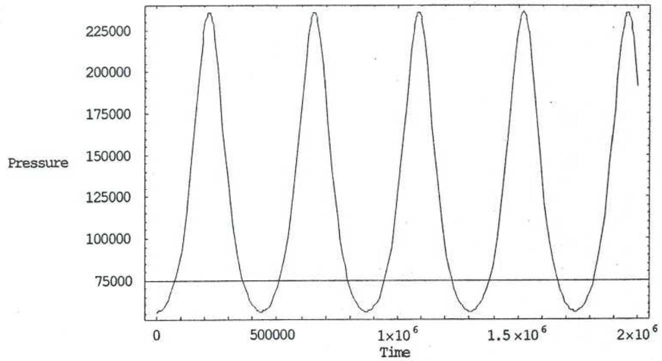
$\alpha = \frac{GM}{R_0^3}$

$\beta = \frac{4\pi R_0^2 P_0}{m}$

Mathematica Findings for ΔRadii, Velocity and Pressure







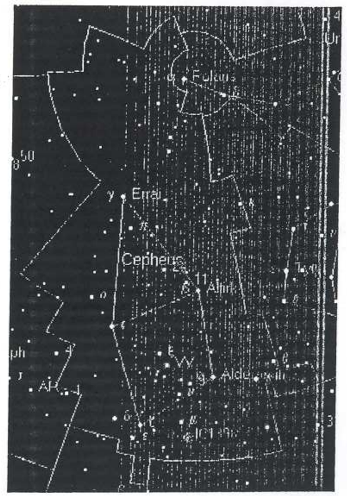
Conclusion

These stars are quite fascinating as the emergence of entire organizations dedicated to their observation would attest. One such organization is the AAVSO, American Association of Variable Star Observers (www.aavso.org).

My guess would be that not many of those observers have had the opportunity to do a similar mathematical study as it is certainly not necessary to appreciate the Cepheid's distinct beauty; however, I would recommend such an exercise to any student of Physics due to the diverse areas of the subject that must be drawn upon in order to get the correct results. It has been a very useful undertaking.

A very popular Cepheid, is Delta Cephei. The primary reason for its popularity is that the entire change in magnitude may be viewed with the naked eye.

Delta Cephei



Credits

- http://www.aavso.org/vstar/vsotm/0900.stm
- http://institute-of-brilliant-failures.com/section3.htm
- http://nedwww.ipac.caltech.edu/level5/ESSAYS/Evans/evans.html
- http://www.aavso.org/
- As well as the various handouts from Professor H.Tahsiri.

$$A = R''[t] + \frac{\alpha}{R[t]^2} - \frac{\beta}{R[t]^3}$$

$$-\frac{\beta}{R[t]^3} + \frac{\alpha}{R[t]^2} + R''[t]$$

$\alpha = 1.4\ 10^{-10}$

1.4×10^{-10}

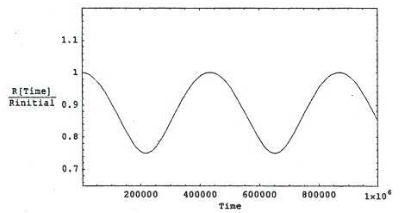
$\beta = 1.2\ 10^{-10}$

1.2×10^{-10}

B = A == 0

$$-\frac{1.2 \times 10^{-10}}{R[t]^3} + \frac{1.4 \times 10^{-10}}{R[t]^2} + R''[t] == 0$$

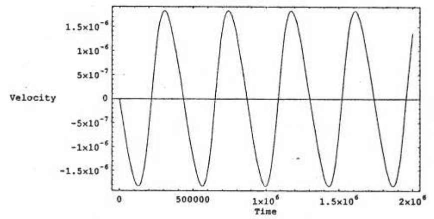
```
Sol = NDSolve[{B, R[0] == 1, R'[0] == 0}, R, {t, 0, 2 10^6}]
{{R → InterpolatingFunction[{{0., 2.×10^6}}, <>]}}
PlotA = Plot[Evaluate[R[t] /. Sol], {t, 0, 2 10^6}, PlotRange → {{2000, 10^6}, {.65, 1.2}},
    Frame → True, FrameLabel → {Time, R[Time]/Rinitial}, RotateLabel → False];
```




```
In[17]:= T = 2π / Sqrt[(3γ - 4) GM/R^3];
In[18]:= γ = 5/3;
In[19]:= G = 6.67 10^-11;
In[20]:= Rsun = 6.96 10^8;
In[21]:= R = 30 * Rsun
Out[21]= 2.088×10^10
In[22]:= Msun = 1.989 10^30;
In[23]:= M = 10 Msun
Out[23]= 1.989×10^31
In[24]:= Tsec = T
Out[24]= 520470.
In[25]:= Tdays = Tsec/86400
Out[25]= 6.02395
```



```
PlotB = Plot[Evaluate[R'[t] /. Sol], {t, 0, 2 10^6},
    FrameLabel → {Time, Velocity}, RotateLabel → False, Frame -> True];
```




```
Pinitial = 5.6 10^4;
P[t_] = Pinitial R[t]^-5
56000./R[t]^5
PlotC = Plot[Evaluate[P[t] /. Sol], {t, 0, 2 10^6},
    FrameLabel → {Time, Pressure}, RotateLabel → False, Frame → True];
```