Math 444 Fall 2006
Newberger

Tips and Remarks for Homework Section 6.1

I. Page 142 #21 In this problem, you are asked to verify that two finite sets are ideals. To do this, check that they are closed under subtraction and closed under multiplication by elements of the ring. To avoid too much tedium, either being by explaining that in each parts, $I = \{3r|r \in R\}$, where R is the ring given in that part.

If you are doing such a problem, and you do not see how to write the set you are studying in set builder notation, use multiplication and subtraction tables to make the computations more concise. You should include a sentence explaining what the entries in the table mean, like, "The entry in the table in row a column b is $a - b$." The multiplication table required to verify that a set is an ideal should have all of the ring elements on one side and the elements of the ideal along the other side. After writing the table, conclude that since all of the entries in the tables belong to the set in question, the set is closed under subtraction and multiplication by elements of the ring.

II. Page 143 #25 This exercise is Theorem 6.10.

III. Page 143 #30 (a) Prove that the set J of all polynomials in $\mathbb{Z}[x]$ whose constant term is divisible by 3 is an ideal.

Use Theorem 6.1.

(b) Show that J is not principal.

Proceed by contradiction. Suppose that J is principal. Then

$$J = \{f(x)p(x)|f(x) \in \mathbb{Z}[x]\}$$

for some $p(x) \in \mathbb{Z}[x]$. Explain that the polynomials $g(x) = 3$ and $h(x) = x$ are elements of J, since they have constant terms divisible by 3. Use the fact that $J = \{f(x)p(x)|f(x) \in \mathbb{Z}[x]\}$ to write $g(x) = 3$ and $h(x) = x$ in terms of $p(x)$ and look for a contradiction.

Note that $p(x)$ is an element of J because $\mathbb{Z}[x]$ is a ring with identity, so there exists $f(x) \in \mathbb{Z}[x]$ such that $p(x) = f(x)p(x)$, namely the constant polynomial $f(x) = 1$. If the ring you are studying does not have an identity, then the ideal generated by that ring will not contain the generator. For example in the ring of even integers, the ideal generated by 2 is $\{2n|n \text{ is even}\} = \{4n|n \in \mathbb{Z}\}$. This ideal does not contain 2.