

## Exam 2 Review Sheet

This exam will cover Chapter 4, Sections 1 through 4, Chapter 5 and Chapter 6.

I. **Vocabulary.** The following list consists of terms that you may be asked to define, and theorems you may be asked to state.

- Leading coefficient and degree (page 84)
- The Division Algorithm in  $F[x]$  (page 85)
- Divides, factor (page 90)
- Greatest common divisor (page 91)
- Relatively prime (page 93)
- Irreducible, reducible (page 95)
- Root (page 101)
- The Remainder Theorem and The Factor Theorem (page 102)
- Congruence in  $F[x]$  (page 119)
- Congruence class (page 120)
- Extension field (page 129)
- Ideal (page 135)
- Principal ideal generated by  $c$  (page 137)
- Congruence modulo  $I$  (page 138)
- Congruence class of  $a$  modulo  $I$  (page 140)
- The quotient ring  $R/I$  (page 140; see also page 147)
- The kernel of a homomorphism (page 148; the definition of  $K$  is given on page 147 in Theorem 6.10)
- The First Isomorphism Theorem (page 149)
- Prime (page 154)
- Maximal (page 156)

II. **Theorems.** You should know the statements of the lemmas, theorems and corollaries found in these sections, so that you may use them in your arguments. You will be asked to provide a proof of at least one of the following (the statements will be given):

- (a) The following part of Theorem 4.13: Let  $F$  be a field. Every nonconstant polynomial  $f(x)$  in  $F[x]$  is a product of irreducible polynomials in  $F[x]$ . (You will not be asked to prove uniqueness.)
- (b) Theorem 5.10. (There are notes in the book that do not constitute a complete proof. You should modify the proof of Theorem 2.8 for  $F[x]$ , as we did in the class notes.)
- (c) Theorem 6.11.

III. **Guarantees.** Your exam will have from 4 to 6 problems, some consisting of several parts.

- (30%-40% of the grade) You will be asked to state some of the definitions in part (I) above and to prove at least one of those listed in (II).
- You will be asked to prove statements, answer questions and provide examples involving the material on the lists above. The proofs will be in the style of the homework problems, involving the material listed in (1) and (2) above. In particular:
  - You will be asked to prove things about particular polynomials or their congruence classes. For example, that two given polynomials are relatively prime, that a given polynomial is or is not irreducible, or that the congruence class of a given polynomial is or is not a unit.
  - You may be asked to prove that two rings are isomorphic, where one of the rings is a quotient ring. You may be asked about the kernel of a homomorphism and its relationship to the homomorphism's image.
  - You will be asked questions about examples of ideals. For example, you may be asked to prove a subset of a ring is an ideal, or to prove a given ideal is or is not prime and or maximal.